Mathematical Model of Instability Phenomenon in Homogeneous Porous medium in vertical downward direction

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Abstract: *This paper deals with Instability (fingering) phenomena arising in two immiscible fluids (oil and water) flow through homogeneous porous media when water is injected in vertical downward direction. The governing equation is nonlinear partial differential equation which has been solved in terms of quadratic polynomial by using generalized separable method with appropriate initial and boundary conditions. The numerical and graphical presentations of solution have been obtained by using MATLAB.*

Key Words: *Porous media, porous matrix, Fluid, MATLAB.*

1. INTRODUCTION:

It is well-known that when a fluid flowing through a porous media is displaced by another fluid of lesser viscosity then instead of regular displacement of whole front, protuberances (fingers) take place which shoot through the porous media at a relatively very high speed. This phenomenon is called **instability (fingering).**

When one fluid is filled in the pores of the homogeneous porous media and another fluid is injected which is not mixing with the residential fluid in ordinary condition then instability occurs in the flow due to the viscosity difference and wettability of those two fluids. It has great importance in petroleum reservoir when oil is recovered by injecting water in the oil formatted region. As a consequence, oil is displaced from the oil formatted region. The fingering is a well-known phenomenon in such type of system. Especially it is useful in secondary oil recovery process when residual oil remains in oil formatted region after primary oil recovery process.

During secondary oil recovery process, when water is injected in oil formatted area, water will flow through interconnected capillaries which shoot through small capillaries to push oil towards the oil production well. Due to external injecting force, irregular fingers are generated and instability occurs in flow. The objective of the study is to find the saturation of injected water occupied by these irregular fingers.

This paper deals with Instability (fingering) phenomena arising in two immiscible fluids (oil and water) flow through homogeneous porous media when water is injected in vertical downward direction with considering mean capillary pressure and gravitational effect. Many researchers have considered injection of water in horizontal direction. Here it is consider that water is injected in vertical downward direction in oil formatted region. When water and oil are flowing in vertical downward direction, the gravitational effect plays important role to increase the velocity of water and oil which has been considered in Darcy's law.

The governing equation of the phenomenon is nonlinear partial differential equation which has been solved in terms of quadratic polynomial by using generalized separable method with appropriate initial and boundary conditions. It has been concluded that saturation of injected water increases as depth z increases for any time $t > 0$ which is fact. It is also concluded that saturation of water increases more due to gravitational effect in vertically downward direction. The numerical and graphical presentations of solution have been obtained by using MATLAB.

The purpose of the study is to find out the saturation of injected water occupied by irregular fingers. Here the fingers of different shape and size are developed due to external injecting force and gravitational effect. To find out the saturation of injected water occupied in different fingers, as suggested by Scheidegger [11], we have considered cross-sectional area occupied by the rectangular schematic fingers of average length which gives the saturation of injected water for given depth (z) and for time (t) > 0.

2. STATEMENT OF THE PROBLEM:

Here it is considered that there is uniform water injection into oil saturated homogeneous porous matrix in vertically downward direction. For the sake of mathematical study, we have chosen a vertical cylindrical piece of homogeneous porous matrix from real field such that it is surrounded by impermeable surface except its two ends (top which is known as common interface (z = 0) and bottom which is connected to oil production well), so that oil and water can flow only in vertically downward direction. When water is injected in homogeneous porous medium in vertical downward direction, oil is displaced by water and instead of regular displacement of the common interface (z = 0), protuberances (fingers) take place which shoot through the interconnected capillaries at a relatively very high speed resulting into instability occurs in the flow. Since oil and water flowing through homogeneous porous media, Darcy's law is applicable for the flow of oil and water. The gravitational effect increases velocity of injected water and native oil in cylindrical porous matrix which is considered in Darcy's law. Due to external injecting force and gravitational effect, the irregular fingers are developed in downward direction, so that water shoots through the interconnected capillaries in vertical downward direction to drag oil towards the bottom of cylindrical piece of porous matrix as shown in the Figure 1.

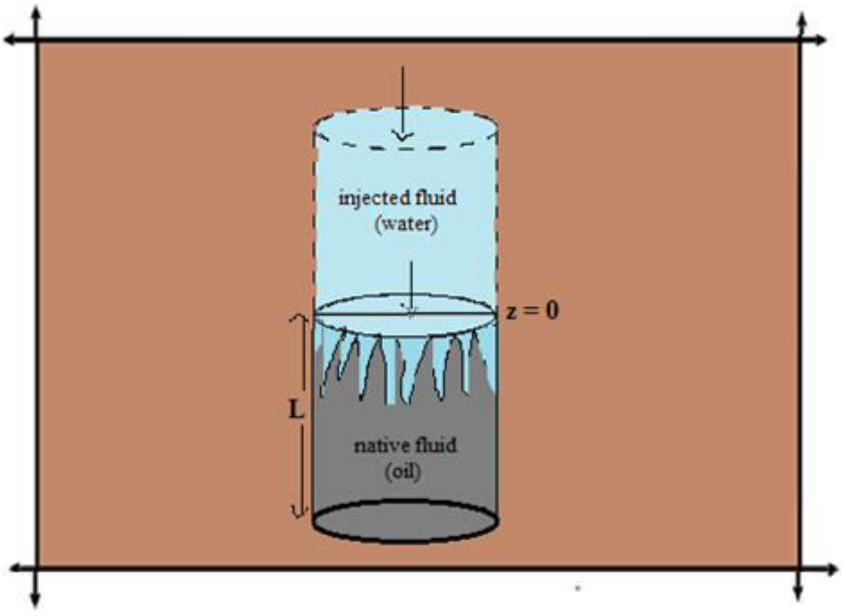

Figure 1: Formation of instability (fingers) in cylindrical piece of porous matrix in vertical downward direction

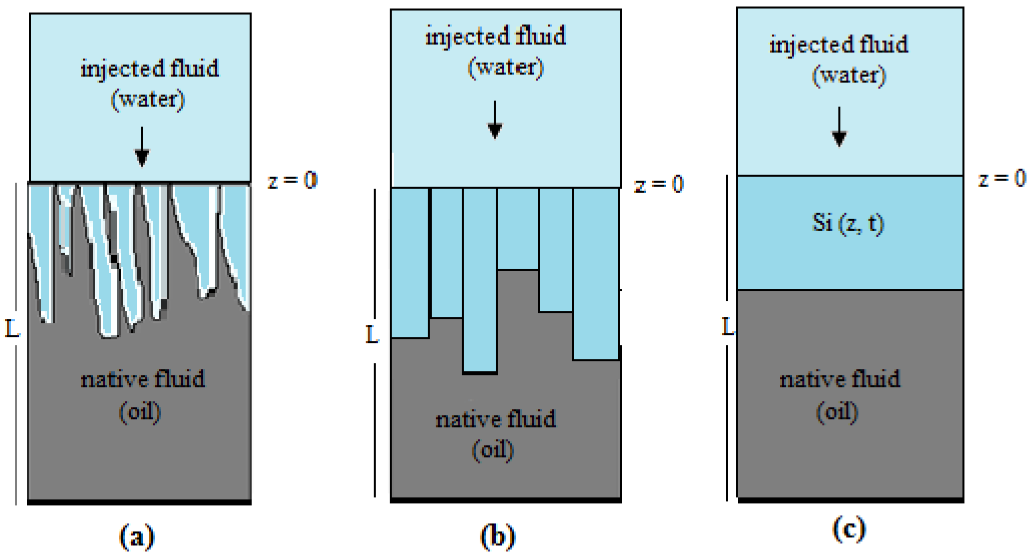

Figure 2: Cross sectional view of instability phenomenon in vertical downward direction

To understand the phenomenon of instability in one dimension, we consider the cross section of chosen vertical cylindrical piece of porous matrix which is rectangle as shown in figure (2-a). The shape and size of fingers are irregular. Therefore for mathematical formulation, we consider schematic fingers as rectangular fingers of different size as shown in figure (2-b). But still it is difficult to find saturation of injected water for any depth z for any given time t > 0. So we choose average cross section area of all rectangular fingers of average length as shown in figure (2-c). In this case the saturation of the injected water (S_i) is defined as average cross-sectional area occupied by schematic fingers at depth z for given $t \geq 0$. If the displacement processes are in a z-direction with injection time t hence this saturation is measured as $S_i(z,t)$ which is function of depth (z) and time (t).

Since water and oil are flowing in homogeneous porous medium, for low Reynolds's number, the Darcy's law is applicable to measure the velocity (V_i) of injected water and velocity (V_n) of native oil. The porous medium is considered homogeneous, so porosity (P) and permeability (K) are considered as constants.

When water and oil are flowing in vertical downward direction, the gravitational effect plays important role to increase the velocity of water and oil by additional term ρg in Darcy's law. [2]

Let the bottom of vertical cylindrical piece of homogeneous porous matrix is at z = L which is measured from top z = 0.

3. MATHEMATICAL FORMULATION:

Since injected water and oil flow through vertical homogeneous porous media, Darcy's law is applicable to each fluid. According to [2], [8] and [11] the volume flux of the injected water V_i and native oil V_n can be described by the Darcy's equation due to gradient in the pressure of injected water P_i and pressure of native oil P_n as follows:

$$V_i = -\left(\frac{K_i}{\mu_i}\right) K \left(\frac{\partial P_i}{\partial z} + \rho_i g\right) \quad (1)$$

and

$$V_n = -\left(\frac{K_n}{\mu_n}\right) K \left(\frac{\partial P_n}{\partial z} + \rho_n g\right) \quad (2)$$

where K is the constant permeability for the homogeneous medium, μ_i *and* μ_n are constant kinematic viscosity of injected water and native oil respectively, ρ_i *and* ρ_n are the density of injected water and native oil respectively, g is the acceleration due to gravity which is constant, K_i and K_n are relative permeabilities of injected water and native oil which are functions of their saturations respectively. The relative permeabilities describe the impairment of the one fluid by the other. The coordinate z is measured along the vertical axis in downward direction of cylindrical porous medium, the origin being at the common interface (z = 0).

When water is injected at common interface z = 0 in downward direction as shown in figure (1) then water will flow through interconnected capillaries and due to injected force oil will flow through homogeneous porous media in downward direction. Hence by conservation of mass for two immiscible and incompressible fluids (water and oil) with constant fluid densities in a homogeneous one dimensional porous medium leads to the following equations known as equations of continuity [1].

$$P\left(\frac{\partial S_i}{\partial t}\right) + \frac{\partial V_i}{\partial z} = 0 \quad (3)$$

$$P\left(\frac{\partial S_n}{\partial t}\right) + \frac{\partial V_n}{\partial z} = 0 \quad (4)$$

where S_i is saturation of injected water, S_n is saturation of oil, P is the porosity of the homogeneous porous medium which is considered constant.

It is well known fact that sum of saturation of water and oil is fully saturated (Unity). Hence we can define for fluid saturation of water and oil as

$$S_i + S_i = 1 \quad , \quad [11] \quad (5)$$

The fluid can flow through interconnected capillaries only due to capillary pressure (P_c) which is defined as the pressure difference of the flowing fluid across their common interface is a function of fluid saturation. It may be written as

$$P_c(S_i) = P_n - P_i \quad , \quad [11] \quad (6)$$

Mehta [81] expressed the linear relationship between capillary pressure (P_c) and saturation of injected water (S_i) as

$$P_c = -\beta S_i \quad (7)$$

where β is constant of proportionality

There are different relationship between permeability and saturation but for definiteness of mathematical analysis, we use standard relations between saturation of water, saturation of oil and relative permeability of water and oil, given by Scheidegger and Johnson [122] as follows:

$$K_i = S_i \quad , \quad K_n = 1 - \alpha S_i \quad (\alpha = 1.11) \tag{8}$$

For more definiteness, we choose $\alpha \approx 1$ then

$$K_n \approx 1 - S_i \;=\; S_n \;(\because S_i + S_n = 1),\; [11] \tag{9}$$

The equations of motion for saturation of injected water and displaced oil can be obtained by substituting the values of V_i and V_n from equations (1) and (2) in to the equations (3) and (4) respectively, we get

$$P\left(\frac{\partial S_i}{\partial t}\right) = \frac{\partial}{\partial z}\left[\frac{K_i}{\mu_i}K\frac{\partial P_i}{\partial z}\right] - \frac{\partial}{\partial z}\left[\frac{K_i}{\mu_i}K\,\rho_i g\right] \tag{10}$$

and

$$P\left(\frac{\partial S_n}{\partial t}\right) = \frac{\partial}{\partial z}\left[\frac{K_n}{\mu_n}K\frac{\partial P_n}{\partial z}\right] - \frac{\partial}{\partial z}\left[\frac{K_n}{\mu_n}K\,\rho_n g\right] \tag{11}$$

On substituting value of P_i from (6) into (10), we get

$$P\left(\frac{\partial S_i}{\partial t}\right) = \frac{\partial}{\partial z}\left[\frac{K_i}{\mu_i}K\left(\frac{\partial P_n}{\partial z} - \frac{\partial P_c}{\partial z}\right)\right] - \frac{\partial}{\partial z}\left[\frac{K_i}{\mu_i}K\,\rho_i g\right] \tag{12}$$

Now from relation (5), we can write

$$\left(\frac{\partial S_n}{\partial t}\right) + \left(\frac{\partial S_i}{\partial t}\right) = 0$$

Hence $\dfrac{\partial S_n}{\partial t} = -\dfrac{\partial S_i}{\partial t}$

Then substituting the values of $\left(\frac{\partial S_n}{\partial t}\right)$ in (11) and equate with (10) we get,

$$\frac{\partial}{\partial z}\left[\left(\frac{K_i}{\mu_i} + \frac{K_n}{\mu_n}\right)K\frac{\partial P_n}{\partial z} - \frac{K_i}{\mu_i}K\frac{\partial P_c}{\partial z}\right] = \frac{\partial}{\partial z}\left[\left(\frac{K_i}{\mu_i}\rho_i + \frac{K_n}{\mu_n}\rho_n\right)Kg\right] \tag{13}$$

Now integrating with respect to z keeping t as constant, we get

$$\left(\frac{K_i}{\mu_i} + \frac{K_n}{\mu_n}\right)K\frac{\partial P_n}{\partial z} - \frac{K_i}{\mu_i}K\frac{\partial P_c}{\partial z} = \left(\frac{K_i}{\mu_i}\rho_i + \frac{K_n}{\mu_n}\rho_n\right)Kg + C(t) \tag{14}$$

where C(t) is the constant of integration which can be determine.

On simplifying the above equation, we get

$$\frac{\partial P_n}{\partial z} = \frac{\frac{K_i}{\mu_i}\frac{\partial P_c}{\partial z} + \left(\frac{K_i}{\mu_i}\rho_i + \frac{K_n}{\mu_n}\rho_n\right)g + \frac{\mathrm{C(t)}}{\mathrm{K}}}{\left(\frac{K_i}{\mu_i} + \frac{K_n}{\mu_n}\right)} \tag{15}$$

Substituting the value of $\frac{\partial P_n}{\partial z}$ from equation (15) into (12), we get

$$\left(\frac{\partial S_i}{\partial t}\right) = \frac{\partial}{\partial z}\left[\frac{\frac{K_i}{\mu_i}K\left(\left(\frac{K_i}{\mu_i}\rho_i + \frac{K_n}{\mu_n}\rho_n\right)g + \frac{\mathrm{C(t)}}{\mathrm{K}} - \frac{\partial P_c}{\partial z}\left(\frac{K_n}{\mu_n}\right)\right)}{\left(\frac{K_i}{\mu_i} + \frac{K_n}{\mu_n}\right)}\right] - \frac{\partial}{\partial z}\left[\frac{K_i}{\mu_i}K\,\rho_i g\right] \tag{16}$$

Now the oil pressure (P_n) as expressed in section- I, in terms of average pressure $\bar{P}$ and capillary pressure P_c as, [13]

$$P_n = \frac{P_n + P_i}{2} + \frac{P_n - P_i}{2} = \bar{P} + \frac{1}{2}P_c \tag{17}$$

where $\bar{P}$ is constant mean pressure of injected water and displaced native oil.

Hence

$$\frac{\partial P_n}{\partial z} = \frac{1}{2}\frac{\partial P_c}{\partial z} \tag{18}$$

Eliminating $\frac{\partial P_n}{\partial z}$ from (18) to (15), we get

$$\frac{1}{2}\frac{\partial P_c}{\partial z}\left(\frac{K_n}{\mu_n} - \frac{K_i}{\mu_i}\right) - \left(\frac{K_i}{\mu_i}\rho_i + \frac{K_n}{\mu_n}\rho_n\right)g = \frac{\mathrm{C(t)}}{\mathrm{K}} \tag{19}$$

Hence using value of $\frac{C(t)}{K}$ in equation (16), we get

$$P\left(\frac{\partial S_i}{\partial t}\right) = \frac{\partial}{\partial z}\left[\frac{K_i}{\mu_i}K\left(-\frac{1}{2}\frac{\partial P_c}{\partial z}\right)\right] - \frac{\partial}{\partial z}\left[\frac{K_i}{\mu_i}K\,\rho_i g\right] \tag{20}$$

For more simplification, using the standard relations (7) and (8) in the above equation (20), we get

$$P\left(\frac{\partial S_i}{\partial t}\right) - \frac{\beta}{2}\frac{K}{\mu_i}\frac{\partial}{\partial z}\left[\left(S_i\frac{\partial S_i}{\partial z}\right)\right] + \frac{K\,\rho_i g}{\mu_i}\frac{\partial S_i}{\partial z} = 0 \tag{21}$$

Which is nonlinear partial differential equation governing the phenomenon of instability when water is injected in downward direction.

Now to reduce the equation (21) in dimensionless form, we choose dimensionless variables as

$$T = \frac{K\beta}{2L^2 P\mu_i}t \quad , \quad Z = \frac{z}{L} \quad , \; 0 \le Z \le 1 \; , \; 0 \le T \le 1$$

where L is the length of the cylindrical porous matrix measured from $z = 0$ to bottom $z = L$.

Therefore, the equation (21) is converted into dimensionless form as

$$\frac{\partial S_i}{\partial T} = S_i\frac{\partial^2 S_i}{\partial Z^2} + \left(\frac{\partial S_i}{\partial Z}\right)^2 - A\frac{\partial S_i}{\partial z} \; , \tag{22}$$

where $A = \frac{2L^2 P\rho_i\, g}{\beta}$ is considered as constant. Since $L, P, \rho_i\,, g$ and β are constants.

To solve this equation (22), it is necessary to use appropriate initial and boundary conditions suitable to the phenomenon of instability.

When water is injected at top (common interface z = 0), we assume that the initial saturation of injected water will be very small. Let the initial saturation of injected water can be expressed as

$S_i(Z,0) = S_0(Z)$, when $T = 0$ and $Z > 0$ (23)

Let the saturation of injected water at common interface (Z=0) and at bottom (Z=L) be $S_{i0}\,(T)$ and $S_{i1}\,(T)$ respectively then a set of boundary conditions can be written as

$S_i(0,T) = S_{i0}\,(T)$, $T > 0$ (24)

and

$S_i(1,T) = S_{i1}\,(T)$, $T > 0$ (25)

respectively. Where L is the total length of the cylindrical porous matrix.

4. SOLUTION BY GENERALIZED SEPARABLE METHOD:

The governing equation (22) is a non linear partial differential equation of parabolic type and it is difficult to obtain exact classical solution. So here we apply generalized separable method to find an analytical approximate solution of (22) along with conditions (23) to (25) in terms of quadratic polynomial of Z.

Hence, the solution of equation (22) can be expressed as quadratic polynomial in Z as follows:

$S_i(Z,T) = \emptyset(T)Z^2 + \Psi(T)Z + \tau(T)$ [4] (26)

where $\emptyset(0)$, $\Psi(0)$ and $\tau(0)$ are non zero constants.

As per [14], the functions $\emptyset(T)$, $\Psi(T)$ and $\tau(T)$ can be determined by a system of first order ordinary differential equations with variable coefficients. Hence using (26) and substituting the values of $\frac{\partial S_i}{\partial T}, \frac{\partial S_i}{\partial Z}, \frac{\partial^2 S_i}{\partial Z^2}$ in equation (22) and equating the coefficients of like powers of Z, we get the system of first order ordinary differential equations with variable coefficients as follows:

$$\emptyset'(T) = 6\emptyset^2 \tag{27}$$

$$\Psi'(T) = 6\emptyset\Psi - 2A\emptyset \tag{28}$$

$$\tau'(T) = 2\emptyset\tau + \Psi^2 - A\Psi \tag{29}$$

The solutions of equations (27), (28) and (29) are

$$\emptyset(T) = -\frac{1}{6T + C_1} \quad , \quad \Psi(T) = \frac{C_2}{6T + C_1} + \frac{A}{3} \quad and$$

$$\tau(T) = \frac{-(C_2)^2}{4(6T+C_1)} + \frac{C_3}{(6T+C_1)^{1/3}} - \frac{AC_2}{3} - \frac{A^2(6T+C_1)}{18} \tag{30}$$

respectively.

Where C_1, C_2 *and* C_3 are constants of integration such that $\emptyset(0),\ \Psi(0),\ \tau(0) \neq 0$

To use initial condition (23), putting $T = 0$ in the solution (26) we get

$$S_0(Z) = \emptyset(0)Z^2 + \Psi(0)Z + \tau(0) \tag{31}$$

where $\emptyset(0),\ \Psi(0),\ \tau(0) \neq 0$ constants.

Let $\emptyset(0) = a\ , \Psi(0) = b,\ \tau(0) = c$ where a, b and c are arbitrary non zero constants.

Now to find C_1, C_2 *and* C_3, substituting the values of $\emptyset(0),\ \Psi(0)$ *and* $\tau(0)$ in (30), we get

$$C_1 = -\frac{1}{a}\ , \qquad C_2 = \frac{1}{a}\left(\frac{A}{3} - b\right)\ , \qquad C_3 = \left(c + \frac{A^2}{36a} - \frac{Ab}{6a} - \frac{b^2}{4a}\right)\left(-\frac{1}{a}\right)^{\frac{1}{3}}$$

Substituting these values of C_1, C_2 *and* C_3 in (30), we get

$$\emptyset(T) = -\frac{1}{6T - \frac{1}{a}}$$

$$\Psi(T) = \frac{\frac{1}{a}\left(\frac{A}{3} - b\right)}{6T - \frac{1}{a}} + \frac{A}{3}$$

$$\tau(T) = \frac{-\frac{1}{a^2}\left(\frac{A}{3} - b\right)^2}{4\left(6T - \frac{1}{a}\right)} + \frac{\left(c + \frac{A^2}{36a} - \frac{Ab}{6a} - \frac{b^2}{4a}\right)\left(-\frac{1}{a}\right)^{\frac{1}{3}}}{\left(6T - \frac{1}{a}\right)^{1/3}} - \frac{A}{3a}\left(\frac{A}{3} - b\right) - \frac{A^2\left(6T - \frac{1}{a}\right)}{18}$$

Now substituting above values of $\emptyset(T),\ \Psi(T)$ *and* $\tau(T)$ in (26), we get

$$S_i(Z,T) = -\left(\frac{1}{6T - \frac{1}{a}}\right)Z^2 + \left(\frac{\frac{1}{a}\left(\frac{A}{3} - b\right)}{6T - \frac{1}{a}} + \frac{A}{3}\right)Z + \left(\frac{-\frac{1}{a^2}\left(\frac{A}{3} - b\right)^2}{4\left(6T - \frac{1}{a}\right)} + \frac{\left(c + \frac{A^2}{36a} - \frac{Ab}{6a} - \frac{b^2}{4a}\right)\left(-\frac{1}{a}\right)^{\frac{1}{3}}}{\left(6T - \frac{1}{a}\right)^{1/3}} - \frac{A}{3a}\left(\frac{A}{3} - b\right) - \frac{A^2\left(6T - \frac{1}{a}\right)}{18}\right) \text{where } A = \frac{2L^2 P \rho_i\, g}{\beta} \tag{32}$$

which is the required solution of governing non linear partial differential equation (22) representing saturation of fingers of injected water in downward direction during instability phenomenon. The numerical values and graphical presentation of the solution (32) can be obtained by MATLAB coding.

5. NUMERICAL AND GRAPHICAL PRESENTATION:

Here for numerical simulation, we consider the following values:

According to [7], the initial saturation of injected water is exponentially decreasing in some special case such that $S_0(Z) = e^{-Z}\ for\ any\ Z > 0.$

Now substituting $S_0(Z) = e^{-Z},\ \emptyset(0) = a\ , \Psi(0) = b,\ \tau(0) = c$ in (31), we get

$e^{-Z} = aZX^2 + bZ + c$

Using expansion e^{-Z} in left hand side and equating the coefficients of like powers of Z (by neglecting Z^3 and higher powers of *Z*),, we obtain the values of *a*, *b* and *c* as follows:

$a = 0.5,\ b = -1,\ c = 1$

The values of some constants are taken from standard literature as below:

$L = 1,\ P = 0.5,\ g = 9.8,\ \rho_i = 0.1,\ \beta = 1 \ \Rightarrow A \approx 1$

The Numerical and graphical presentations of solution (32) have been obtained by using MATLAB coding. Figure 3 shows the graphs of S_i Vs. Z for given fix time T= 0.5, 0.6, 0.7, 0.8 and Table-4.2 represent the numerical values for S_i Vs. Z for given fix time T= 0.5, 0.6, 0.7, 0.8. The following table shows the saturation of injected water (S_i) for different Z for fixed T > 0.

Time (T) ➔	0.5	0.6	0.7	0.8	0.9

Depth (Z) ↓	Saturation of injected water (S_i)				
0	0.0933	0.0941	0.0950	0.0958	0.0966
0.1	0.1054	0.1070	0.1086	0.1103	0.1119
0.2	0.1255	0.1295	0.1335	0.1375	0.1414
0.3	0.1535	0.1614	0.1694	0.1774	0.1854
0.4	0.1894	0.2030	0.2165	0.2300	0.2436
0.5	0.2333	0.2540	0.2747	0.2954	0.3161
0.6	0.2852	0.3146	0.3441	0.3735	0.4030
0.7	0.3450	0.3848	0.4246	0.4644	0.5042
0.8	0.4128	0.4645	0.5162	0.5679	0.6197
0.9	0.4885	0.5538	0.6190	0.6842	0.7495
1	0.5722	0.6525	0.7329	0.8132	0.8936

Table 1: Saturation of injected water (Si) during instability phenomenon for different Z for fixed time T > 0

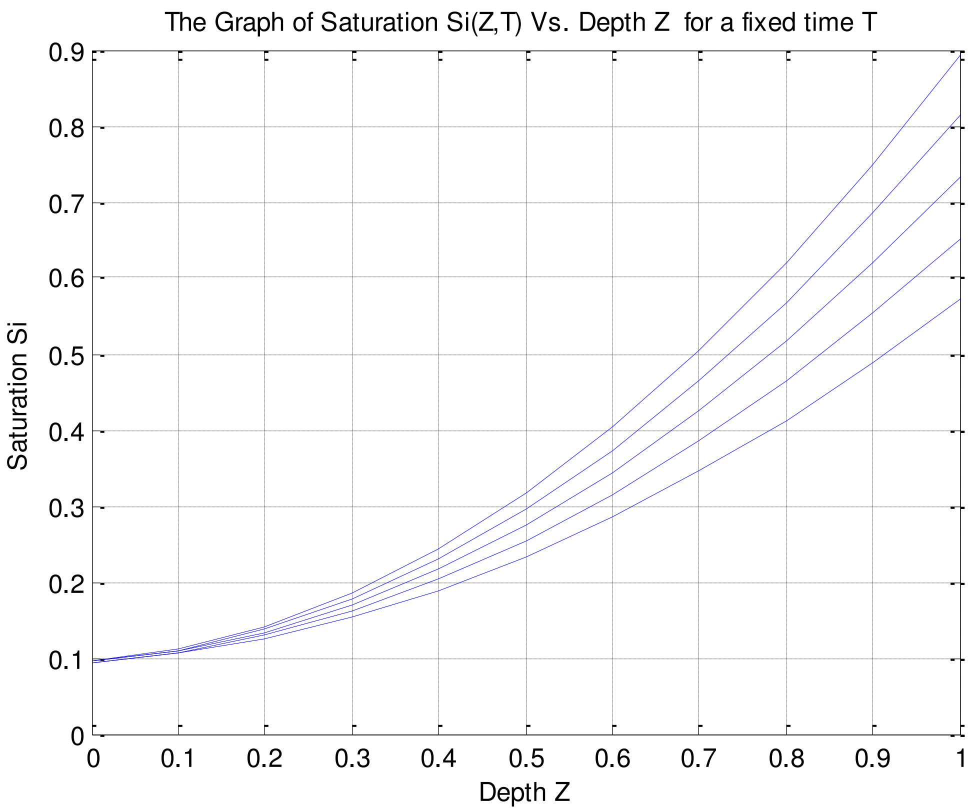

Figure 3: Graph of saturation of injected water ($Si_)$ Vs. depth (Z) for fixed time T > 0

6. INTERPRETATION AND CONCLUSION:

The solution (32) represents Saturation of injected water in downward direction for any depth Z for any time T > 0 under gravitational effect. It is an average cross sectional area occupied in schematic fingers of any depth Z for T > 0. The saturation of water increases in downward direction when depth Z increases for T > 0. The solution is in the form of quadratic function of Z which satisfies both the boundary conditions (24) and (25) at Z = 0 and at Z = L respectively. The saturation of water increases for any depth Z at any time T > 0 which is also shown graphically in Figure 3 and it is consistent with physical phenomena. As saturation of injected water increases in a downward

direction with respect to depth and time, it will displace oil from the oil formatted region towards the bottom of the cylindrical porous matrix. Thus remaining oil can transferred at the bottom during secondary oil recovery process.
The graphical representation and numerical values are given using MATLAB. The graph of S_i vs. Z for any time T is steadily increasing for depth Z and any given time T > 0 which concludes that the fingers may be stabilized after some depth Z in downward direction.

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