# Fractal Dimension Computational Methods

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***Abstract:*** *The different methods for calculating fractal dimension are Box counting method, walking-divider method, Prism counting method, Epsilon-Blanket method, Perimeter-Area Relationship, Fractional Brownian Motion .*

***Key Words:*** *dimension, method, prism, perimeter.*

## 1. DIFFERENT APPROACHES OF MEASURING FRACTAL DIMENSION

### 1.1. Box –dimension method

In this method the fractal object is covered with a grid of n-dimensional boxes with side lengths $\beta$ and then counting the number of boxes that contain a part of the fractal object, say N($\beta$). The fractal surface is covered with boxes of recursively different sizes. The slope *m* is obtained from the logarithmic plot of the number of boxes used to cover the fractal against the box size and the fractal dimension $D_B$ is given by.

$$D_{B=-m} \qquad (1.1)$$

This dimension is also known as the box or Minkowski dimension. For a smooth one-dimensional curve, it is expected that

$$N(\beta) = \frac{L}{\beta} \qquad (1.2)$$

where L is the length of the curve, N(μ) is the number of non empty boxes and μ is the step size.
The generalization to the box counting measure is given as-

$$N(\beta)\alpha \frac{1}{\mu^{D_B}}$$

$$D_B = lim - \frac{logN(\beta)}{log(\beta)}.$$

The Fractal dimension is given by-

$$D = \frac{\log N(\beta)}{\log \beta} \qquad (1.3)$$

we can calculate the fractal dimension of image in the plane, we begin by covering an area by a grid of different mesh sizes, and then we compare the grid sizes and the number of squares containing at least a part of the image. Then the ratio of the number of grids containing the objects gives us dimension.

Steps involved in Box Counting method-

- First we take grids of diffent mesh sizes
- for each grid mesh size, we count the number of grids that the each image contains.
- Next we Pair all combinations of the counted values
- Then we place these values in the standard equation of the Box Counting method .

The dimension is calculated by

$$\boldsymbol{D = \frac{\log N(b) - logN(a)}{log(\beta_b) - log(\beta_a)}} \qquad \textbf{(1.4)}$$

Where, $N(a)$ = number of squares containing the portion of the image when grid size is $a$ cm.
$N(b)$ = number of squares containing the portion of the image when grid size is b cm.

$$\beta_a = \frac{1}{grid\ size(a)}, \quad \beta_b = \frac{1}{grid\ size(b)}$$

The average of result gives us a good estimation of the objects Fractal dimension.

### 1.2 .Walking-Divider method

In this method steps and lengths are required to measure fractal dimension. chord length *(Step)* is used as steps and the number of chord lengths *(Length)* is used as length to cover a fractal curve. This technique is based on the principle of taking rulers of varying size (*Step)* to cover the curve. We count the number of rulers *(Length)* required in each case.

here,

*N(β) = Length* ,

*β = Step*

A least squares fit is used to the bilogarithmic plot of *Length* against *Step* gives the slope m which is the fractal dimension, D with sign reversed

. $D_B = lim - \frac{logN(\beta)}{log(\beta)}$ (2.1)

The limitation of this method is that the initial and final *Step* must be carefully chosen. An appropriate starting value suggested by Shelberg (1982) is half of the average distance between the points. The computation of the initial value, and the procedure required to count the number of *Steps,* makes this algorithm time consuming.

**1.3. Prism Counting**

This algorithm is similar to box counting technique, in this method we compute the area based on four triangles defined by the corner points followed by summation over a grey level surface. The triangles define a prism based on the elevated corners and a central point computed in terms of the average of the four corners. A bilogarithmic plot of the sum of the prisms' areas for a given base area gives a fit to a line whose slope is m in which

D = 2- m (3.1)

Here D is the fractal dimension. Though the algorithm is similar to the box counting method, it is slower due to the number of multiplications implied by the calculation of the areas.

**1.4. Epsilon-Blanket**

In this method the fractal dimension of curves/surfaces are computed by measuring area or volume. the area/volume measured at different scales (Peleg et al., 1984). In such type of curves, ε is considered as the set of points whose distance from a curve is not more than a small scale,. This gives a strip of width *2*ε that surrounds the curve. The length of the curve *L(*ε*)* is calculated from the strip area *A(*ε*)* by

$L(\in) = \frac{A(\in)}{2}$ **(4.1)**

The fractal dimension, *D* is computed using this relationship and is given by

$\boldsymbol{L}(\in) \propto \in^{(\mathbf{1}-\boldsymbol{D})}$ **(4.2)**

In case of surfaces, the set of points in three-dimensional space which is not more than ∈ from the surface, gives a 'blanket' of volume V(∈) whose width is .again 2∈. The surface area is

given by

$\mathrm{A}(\in) = \frac{\mathrm{V}(\in)}{2\in}$

and *D* can be computed as

$A(\in) \propto \in^{2-D}$ **(4.3).**

**1.5. Perimeter-Area Relationship**

Here, the perimeter L is related to the enclosed area *A* for a non-fractal closed curve in the plane by L = c√*A.*

Here the pre-factor *c* is a constant for a given shape.

For squares *c* = 1 and for circles *c - 2*√*π.*

The above relation is generalized by Mandelbrot (1983) for closed fractal curves

as follows from which the fractal dimension, *D* can be computed as

L = c (√A)D **(5.1)**

For $1 < D < 2$

**1.6. Fractional Brownian Motion**

A fractional Brownian motion , $B_H(t)$, is a function whose increments is given by

$\Delta BH_x = B_H(t+x) - B_H(t)$ (6.1)

has a zero-mean Gaussian distribution with variance given by

$|B_H(t+x) - B_H(t)|^2 \propto |x|(2H)$ (6.2)

(Turner et al.,1998). The scaling behavior is defined for the parameter 0 < H < 1. If a fractional Brownian motion. that covers a time period Δt = 1 is considered and the vertical range $\Delta B_H$ is defined as one, then $B_H(t)$ is statistically self-similar. So if the time span is divided into $N = \frac{1}{(\Delta t)}$ equal intervals, the vertical range within these intervals will be

$\Delta B_H = \Delta the = 1/N^H =_N (-H)$ (6.3)

Using the box counting method, with boxes of length $\delta = N^{-1}$, the number of boxes required to cover each interval is
$\Delta B_H \Delta t = {}_N\text{-}H_{/N}\text{-}1 = {}_N(1\text{-}H)$ (6.4)
which means
N(δ)NN(1-H)=N(2-H) (6.5)
So from equation (2.11) we have,
D = 2-H (6.6)
The same formula can be extended to signals of higher topological dimensions to compute the respective fractal dimension.

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