The Components of Deflection of the Vertical Calculations for Heights in Computations and Adjustments of Physical Projects

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Abstract: *The deflection of the vertical is an important parameter of the local gravity field which must not be neglected during survey measurements. In most survey (Geodetic) measurements, the components of deflection of the vertical are not considered as a result of non availability of detailed maps in such area or region. The components of deflection of the vertical using the GPS/levelling (Geometric) method were determined. Fifteen ancillary stations and one control station were used. The orthometric and the ellipsoidal heights of the stations were determined using digital Level and GPS respectively. The least squares adjustment principle was carried out on the observations with the components as parameters. The values obtained are: -0.550" ± 0.000001" and -0.395" ± 0.0000006" for ξ (north-south) and η (east-west) directions respectively. The components of deflection of the vertical of the study area were also computed using three geopotential models (EGM08, EGM96 and EGM84). They are: ξ = 0.179", η= -0.016; ξ = -0.150", η= 0.006 and ξ = -1.024", η= -1.355 respectively. It is recommended that carrying out physical constructions such as, buildings, roads, drainages, etc. that geoid model which serves as a reference surface for heights determination should be obtained by computing the components of deflection of the vertical.*

Key words: *least squares adjustment, GPS, geometric method, ellipsoidal height, orthometric height.*

1. INTRODUCTION:

According to [1], the deflection of the vertical (ε) is the angle between the direction of the gravity vector *g* or plumb line at a point, and the ellipsoidal normal through the same point for a particular ellipsoid. It is conventionally divided into two perpendicular components; a north–south meridional component (ξ) and an east–west prime vertical component (η) (see Fig.1). The direction of the plumb line and its vertical deflection provide valuable information on the structure of Earth's gravity field. For several decades, astrogeodetic observations were primary gravity field observables and used for astrogeodetic geoid determinations, [2]. It can also be predicted using gravimetric, GPS and levelling methods and the digital Zenith cameras, [3]. Astro-geodetic technique adopts astronomical coordinates (Φ, Λ) and geodetic coordinates (ϕ, λ) while gravimetric technique is based on Stokes formula, using abnormalities of the earth gravitational field as the input data. In addition to the two techniques mentioned above, global geopotential and local gravimetric models as well as combined techniques (GPS/Levelling, GPS/Gravimetric, etc.), are used to attain such figures. Global geoid models, such as EGM 96 are developed by using the gravitational information of the whole world. Geoid heights are calculated by using the potential harmonic coefficients of the global geoid models. On the other hand, local geoid models vary, depending on the geodetic data resources of the country being considered. For regions with no gravity information, the orthometric heights obtained via geometric levelling and the ellipsoidal height obtained via GPS are used in combination.

The deflection of the vertical is an important parameter of the local gravity field and has applications in surveying activities such as in determination of directions (bearings), azimuths, zenith angles and slope distances onto the ellipsoid. A practical example is the determination of water flow which requires the geoid knowledge. The position of the geoid in relation to the reference ellipsoid can be determined not only through geoid height but also by deflection of the vertical at the point of observation. It is well known that physical development such as road construction, drains, dams, tunnels etc require a good knowledge of the geoid, [4]. Ugbowo is a developing area in Benin City which requires a good knowledge of the geoid to enable geometric heights to be converted to practical heights so as to determine suitable gradients/slopes to ensure direction of flow of water. Determining the deflection of the vertical components of the study area, will enable the local geoid model of the study area to be determined.

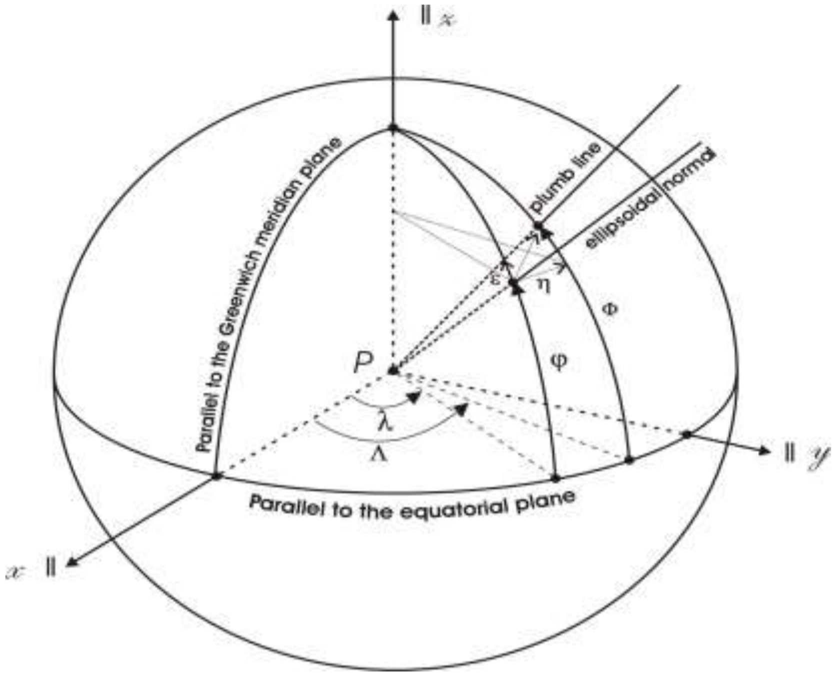

Fig. 1: Deflection of the Vertical Components
Source: Ayhan (2009)

2. AIM AND OBJECTIVES:

The aim of the study is to compute the components of deflection of the vertical of Ugbowo area Benin City, Edo state, using GPS and precise levelling (geometric) method. The primary objectives are:

i. To carry out levelling and GPS observations within the study area so as to evaluate the components of deflection of the vertical.
ii. To determine the components of deflection of the vertical of the study area using three global geopotential models (EGM08, EGM96 and EGM84)

3. STUDY AREA:

The study area is Ugbowo in Benin City. It is a developing area located along Benin Lagos Road in Ovia–North East Local Government Area of Edo State. The Community lies between latitudes $06^0\ 17^1$ N and $06^0\ 25^1$ N and longitudes $05^0\ 34^1$ E and $05^0\ 38^1$ E. It occupies an area of about 2.5 square kilometres with a population of about 10,000 according to 2006 National Population census. Figures 1.1, 1.2 and 1.3 show the location of the study area.

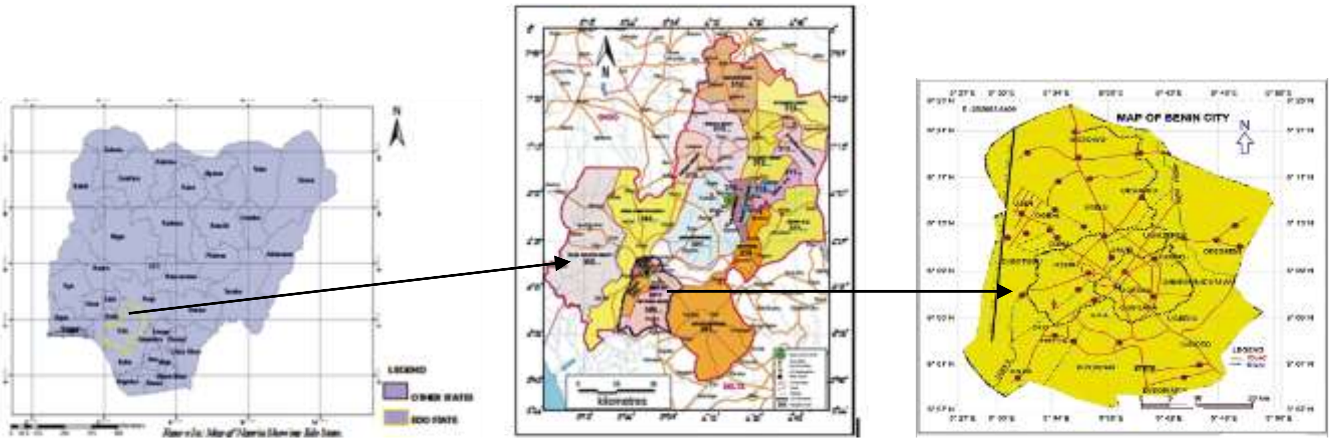

Fig. 1.1: Map of Nigeria Showing Edo State **Fig. 1.2: Map of Edo State Showing Benin City** **Fig. 1.3: Map of Benin City Showing Study Area**
Source: Ministry of Lands and Surveys, Benin City.

The study was limited to the determination of the components of deflection of the vertical of Ugbowo area in Benin City. The scope of the study is as follows:

- Carrying out GPS observation and precise levelling on the chosen points so as to determine their ellipsoidal and orthometric heights respectively.

- Plotting of baselines between the control points and other 15 ancillary stations.
- Evaluation of the components of deflection of the vertical using observation equation method of Least Square Adjustment principle and statistical test of results.
- Computation of components of deflection of the vertical of the study area for EGM08, EGM96 and EGM84 global geopotential models for reference purposes.

4. THEORETICAL MODELS

The differential relationship between geoid height and deflection of the vertical is defined through the following formulae [5], [6]:

$$dN = -\varepsilon . ds \tag{1}$$

or

$$\varepsilon = \frac{-dN}{ds} \tag{2}$$

Deflection of the vertical on any geodetic azimuth (α) direction are resolved and calculated as follows, by using north-south and east-west components (See Fig. 1):

$$\varepsilon = \xi . \cos\alpha + \eta . \sin\alpha \tag{3}$$

When equations (2) and (3) are combined, the following result is obtained;

$$\frac{-dN}{ds} = \xi Cos\alpha + Sin\alpha \tag{4}$$

The differential elements in equation (4) are replaced by the differences obtained in geodetic measurements, and the result is as follows;

$$\frac{-\Delta N}{\Delta s} = \xi Cos\alpha + Sin\alpha \tag{5}$$

On the earth's surface, for any A and B points close to each other, geoid heights are defined in terms of ellipsoidal (h) and orthometric heights (H), using the following equations (6) and (7):

$$N_A = h_A - H_A \tag{6}$$

$$N_B = h_B - H_B \tag{7}$$

Subtraction of equation (6) from equation (7) gives the geoid height difference (ΔN_{AB}) between point A and point B, as follows:

$$\Delta N_{AB} = N_B - N_A = \Delta h_{AB} - \Delta H_{AB} \tag{8}$$

Finally, when equation (8) is substituted into equation (5) for ΔN, equation (9) is obtained;

$$-\frac{\Delta h_{AB} - \Delta H_{AB}}{\Delta S} = \xi Cos\alpha + \eta Sin\alpha \tag{9}$$

Where, ΔH_{AB} is the change in orthometric height between point A and B, and Δh_{AB} is the change in ellipsoidal height between point A and B. In this case, equation (9) is a two-variable equation (ξ and η). In this equation, alpha (α) is calculated through geodetic coordinates measured in points A and B. In order to calculate the deflection of the vertical components for any point A, one requires secondary points, such as B and C. Deflection of the vertical components of point A can be calculated using the ellipsoidal and orthometric heights of point pairs of (A, B) and (A, C)

4.1 Computation of Weighted Matrix

The elements of the weighted matrix are computed from the following equations [7]

$$\sigma_S^2 = \frac{\Delta X \sigma_{\Delta X}^2 + \Delta Y \sigma_{\Delta Y}^2 + \Delta Z \sigma_{\Delta Z}^2}{S_{AB}} \tag{10}$$

where,

$$S_{AB} = \left(\Delta X^2 + \Delta Y^2 + \Delta Z^2\right)^{1/2} \tag{11}$$

$$P_{ii} = \frac{1}{\sigma_S^2} \tag{12}$$

$$P_{ij} = 0 \tag{13}$$

4.2 Computation of Unknown Parameters

The unknown parameters (ξ,η) are computed from least squares principles using observation equation method for the weighted observations, employing equations (14) and (15)

$$X = -(A^T PA)^{-1}(A^T PL) \tag{14}$$

$$X = \begin{pmatrix} \xi \\ \eta \end{pmatrix} \tag{15}$$

where, *A* is the coefficient (design) matrix of the unknown parameters, *L* is the matrix of observations, P is the weight matrix and X is the solution vector matrix.

4.3 Statistical Evaluation

The residuals V are computed after adjustment using the observation equation model (16) [7]

$$V = AX + L \tag{16}$$

The a posteriori variance, σ_o^2 of the adjustment can be computed using equation (17) and noting that a priori variance can be computed from equation (10)

$$\sigma_o^2 = \frac{(V^T PV)}{m-n} \tag{17}$$

The a posteriori standard error, σ_o can be computed using [7]

$$\sigma_o = \sqrt{\frac{(V^T PV)}{m-n}} \tag{18}$$

where *m-n* is the degree of freedom in the adjustment, *m* is the number of observations, *n* is the number of unknown and P is the weight matrix. The standard error of the individual adjusted parameters are obtained from $\sigma_\xi = \sigma_o\sqrt{a_{11}}$ and $\sigma_\eta = \sigma_o\sqrt{a_{22}}$, where a_{11} and a_{22} are the diagonal elements of the covariance matrix [7]. The inverse of the normal matrix and the covariance matrix are respectively given as:

$$N^{-1} = (A^T PA)^{-1} \tag{19}$$

and $$\sum\nolimits_{X_a} = \sigma^2 N^{-1} \tag{20}$$

5. METHODOLOGY:

The methodology was divided into various stages such as data requirements, data acquisition, data processing and data presentation.

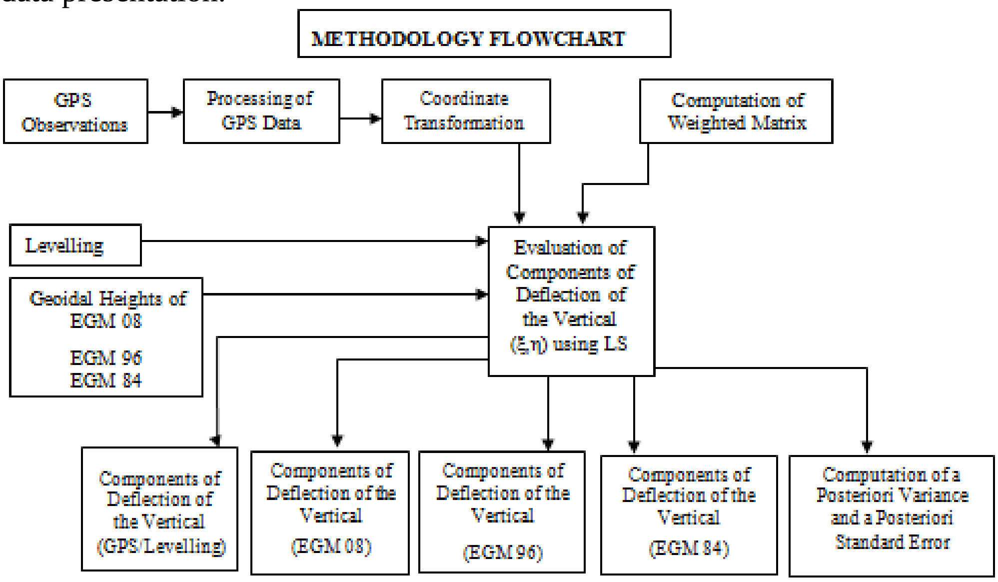

Fig. 2: Flowchart of the Methodology

5.1 Method of Data Processing

5.2 Reconnaissance

The study area was visited and suitable points were chosen and marked with wooden pegs along the road. A nearby control was also located and its coordinates obtained from the Ministry of Lands and Surveys, Benin City.
The digital level was tested using the two-peg-test in order to ensure that it was in good working condition.

5.3 Monumentation

Pre-cast property beacons (Survey Pillars) with dimensions 18cm x 18cm x75cm were used to replace the wooden pegs that were fixed at the selected points during reconnaissance. The pre-cast property beacons consist of good proportion of cements, gravel, sand and water mixed in a ratio of 1:2:3 respectively. Also a number (OOF) template was engraved on the beacons.

5.4 Procedure for Spirit Levelling

The orthometric height of the main control station (AUV 21) was determined with reference to a benchmark (BMK9A) located along Ugbowo Lagos Road. Having obtained the orthometric height of the main control station, the orthometric heights of the ancillary stations were determined with respect to the main station. The levelling of the ancillary stations was divided into three loops (see Fig. 3); the first loop started from AUV21 through OOF 1 to OOF 4 and closed on AUV 21. The second loop started from AUV 21 through OOF 5 to OOF 9 and closed on AUV 21. The third loop also started from AUV 21 through OOF 10 to OOF 15 and also closed on AUV 21.

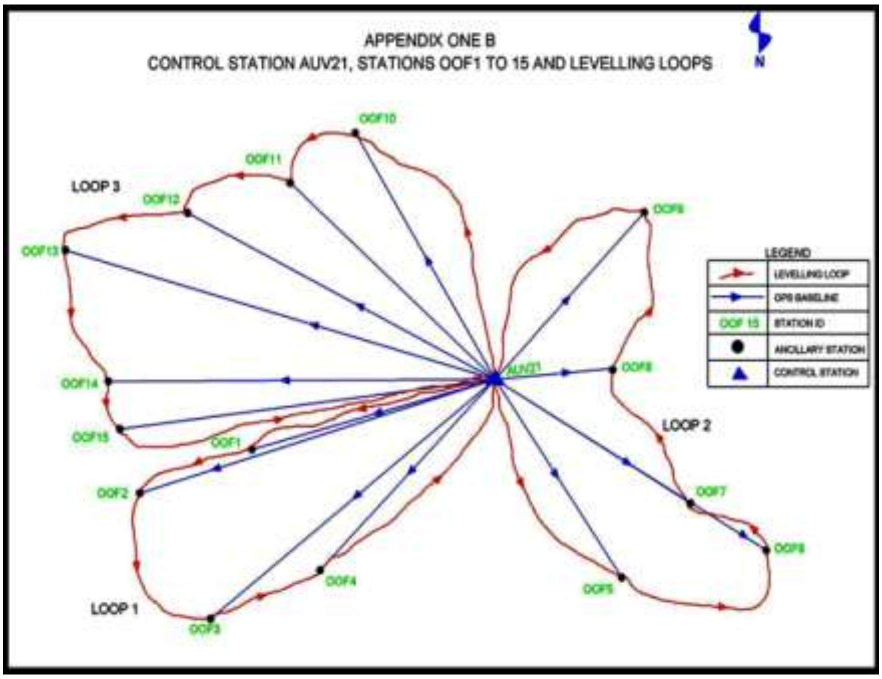

Fig. 3: Control Station AUV 21, Stations OOF1 to OOF15 of Leveling Loops

5.5 Procedure for Differential GPS Observation

The DGPS was used for the acquisition of the 3-D coordinates of the ancillary stations. The observation was carried out in post- processing static mode with the base receiver at the main control station AUV 21 (Fig. 4) and the rover receiver was moving from one of the ancillary stations whose position and ellipsoidal height were to be determined to another.

Fig. 4: Base Receiver at Control Station AUV 21

5.6 Data Processing Procedure

5.7 Processing of Raw Data

The reduced orthometric heights of the ancillary stations and the main control station determined from the spirit levelling were corrected by finding the difference between the known and the observed orthometric height of the main control station of each of the levelling loops. The difference (misclosure) of each loop was divided by the number of instrument stations and the results was multiplied consecutively with the number of stations and added to the reduced levels to obtain the corrected reduced levels of the stations. The corrected reduced levels of the stations were inputted into the computer system in a Microsoft Excel 2010 spreadsheet.

The GPS data was downloaded into a computer system by direct cable connection from the DGPS to the computer system using HcLoader software. The downloaded data was processed and adjusted with Compass software.

5.8 Deduction of the EGM08, EGM96 and EGM84 Geoid Heights

The observed geodetic latitude and longitude of the stations were used online to compute the EGM08, EGM96 and EGM84 geoid heights of the stations. The EGM08, EGM96 and EGM84 geoid heights were calculated using GeoidEval software.

5.9 Computation of Weighted Matrix

The weighted matrix was computed using equations 12 and 13. The variances of the GPS observations were obtained from the variance covariance matrix of the processed observations using compass post processing software. The reciprocal of the variances gave the weights of the observations. The weight matrix of the observations is a diagonal matrix.

5.10 Computation of Deflection of the Vertical Components

The components of deflection of the vertical of the study area were computed using equation (9). The components of deflection of the vertical were also computed using three geopotential models (EGM08, EGM96 and EGM84) for the study area. The model used for the computation of the components of deflection of the vertical of the three geopotential models is given in equation (5). The computation of the deflection of the vertical components was done with online software called Matrix Calculator.

5.11 Statistical Evaluation

The computation of the residuals (*Vs*), the a posteriori variance, a posteriori standard error and the standard error of each of the unknown (components of deflection of the vertical) were carried out using equations (16), (17), (18) and (20) respectively. These were also computed using the Matrix Calculator.

6. RESULT ANALYSIS:

6.1 Analysis of the Spirit Levelling Results

Table 1 shows the results of the closing errors. The observed orthometric heights of the ancillary stations and that of the main control station were seen to be in good shape as shown by the two peg test and the difference between the observed and the known height of the closing station. The closing error for the first loop was 0.0036m, the second loop closing error was found to be 0.0021m and that of the third loop was 0.0042m these were less than 5mm standard hence the results were acceptable.

Table 1: Known and Observed Height of the Closing Stations

Station	Description	$H_{(known)}$ (m)	$H_{(observed)}$ (m)	ΔH (m)
AUV 21	starting and closing station	97.5579	97.5543	0.0036
AUV 21	closing station	97.5579	97.5558	0.0021
AUV 21	closing station	97.5579	97.5537	0.0042

6.2 Analysis of the DGPS Result

The DGPS observations were carried out for about seven hours and were processed as a loop. From the processing result, the observations were seen to have passed both the Network Adjustment and Chi Square Tests which implies that the normal matrix generated during the processing/adjustment was a regular one and inverted accordingly for the calculation of residuals.

6.3 List of Values of the Determined Components of Deflection of the Vertical and those of Global Geopotential Models EGM 08, EGM 96 and EGM 84.

Table 2 shows the computed components of deflection of the vertical. The components of deflection of the vertical of the study area were computed to be -0.549504" and -0.395438" for north-south (ξ) and east-west (η) directions respectively. The standard errors of the computed components of deflection of the vertical were ± 0.000001" and ± 0.0000006". Also the EGM08 components of deflection of the vertical of the study area were computed to be 0.1787" and -0.0156" north-south (ξ) and east-west (η) directions respectively. Those of the EGM96 were computed to be -0.1496" and 0.0061". The components of deflection of the vertical of the study area using EGM84 were also computed to be -1.0243" and -1.3546 north-south (ξ) and east-west (η) directions respectively.

Table 2: Computed and other Available Components of Deflection of the Vertical with Locations

RESEARCHER	LOCATION	METHOD	COMPONENTS OF DEFLECTION OF THE VERTICAL	
			NORTH/SOUTH (ξ)	EAST/WEST (η)
Authors (2016)	Ugbowo, Benin City	GPS/Levelling	-0.550" ± 0.000001"	-0.395"± 0.0000006"
		EGM08	0.179"	-0.016
		EGM96	-0.150"	0.006".
		EGM84	-1.024"	-1.355"
Ameh (2013).	Lobi, Makurdi	GPS/Levelling	-3.18"±0.60"	-2.25"±0.43"
Kantomah (2010)	ABU Zaria	GPS/Levelling	-0.04462"	0.0575856"
Tse and Baki	Hong Kong	GPS/Levelling	-7.3±1.6	5.3±4.3
Ayhan (2009)	Honya, Turkey	GPS/Levelling	-4.15" ±0.61"	8.75" ±0.69"
Tomas (1989)	USA	GPS/Levelling	5.2 ± 0.10"	-2.76 ± 0.14"
Tomas (1989)	USA	Astrogeodetic	5.19 ± 0.5"	-2.58 ± 0.5"

6.4 Statistical Analysis

Table 3 presents the statistics of the computations. The maximum and the minimum residuals were computed to be −0.000205886 and 0.000000219 respectively. The a posteriori variance and the a posteriori standard error were also respectively computed to be 0.182436624" and ± 0.427126". It can also be seen from Table 3 that the standard errors of the determined components of deflection of the vertical are ± 0.008803" and ± 0.005412" for north-south and east-west directions respectively.

Table 3: Statistical Parameters Computed

Parameters	Value in seconds
Residuals: Maximum: Minimum:	0.0007884" -0.7411896"
a posteriori variance	0.182436624"
a posteriori standard error	±0.427126"
Standard error of unknown parameters: σ_{ξ} : σ_{η} :	±0.008803" ±0.005412"

7. CONCLUSION:

The components of deflection of the vertical of Ugbowo area in Benin City was determined using the geometric method; one main control station and fifteen ancillary stations were used. The processed GPS data and orthometric heights of the points were used to compute the components of deflection of the vertical of the study area. The components of deflection of the vertical of the study area were also computed using three global geopotential models (EGM08, EGM96 and EGM84). The a posteriori variance and the a posteriori standard errors were computed using the residuals and the redundant observations. The standard errors of the determined components of the deflection of the vertical for north-south and east-west directions were also computed using the a posteriori variance and the inverse of the normal matrix.

8. RECOMMENDATIONS:

Having determined the components of deflection of the vertical of Ugbowo Area in Benin City, the following recommendations are made:

- Whenever physical construction such as of roads, drains, etc is to be carried out in the study area (Ugbowo), the geoid model which serves as a reference surface for height determination should be obtained from the determined components of deflection of the vertical.
- Whenever astronomic observation in the study area is to be converted to geodetic observation, the determined components of deflection of the vertical should be applied.

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