NUMERICAL SIMULATION OF NATURAL CONVECTION IN RECTANGULAR ENCLOSURES

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Abstract: *The movement of the fluid in natural convection results from the buoyancy forces imposed on the fluid when its density in the proximity of the heat transfer surface is decreased as a result of the heating process. In this work, computational study of the flow initiated by natural convection was done. A numerical study was performed on a rectangular enclosure with varying aspect ratio to predict streamlines and isotherms distributions of a 2-D buoyancy driven turbulent flow. The RANS, energy and turbulent transport equations with Boussinesq approximations were discretized using Finite Difference method. The low Reynolds k-epsilon ($\kappa-\epsilon$) model, Aspect ratio of 2,4,6 and 8 and Prandtl number 0.71 and at a constant Rayleigh numbers (Ra) were used and governing equations were solved. The results of isotherms and streamlines had been represented. Based on the numerical results, as the aspect ratio increased, the flow becomes less turbulent.*

Key Words: *Turbulent Natural convection, Enclosure, Heat transfer, dimensional, Boussinesq.*

1. INTRODUCTION:

Convection is a Mode of heat transfer in fluids. It depends on the fact that fluids expand when heated and thus undergo a decrease in density. As a result, the warmer, less dense *portion* of the fluid will tend to rise through the surrounding cooler fluid.

According to Matthew p, Wilcox (2013) fluid flow can be grouped into two categories, laminar or turbulent flow. In laminar flow, the motion of the fluid particles is very orderly and fluid moves in sheets that slip relative to each other. Turbulence convection is an irregular or disturbed flow. It behaves with a chaotic and unpredictable motion. Beyond a particular temperature difference, the heated fluid rises and cooled fluid falls thereby become turbulent.

The study of natural convection in an enclosure has many engineering applications ranging from simple space heating of domestic rooms to parts of industrial and nuclear installations. For example this kind of flows occurs in building technology, cooling of electronic equipment and material processing.

For the numerical calculation of turbulent flows, an averaging of the Navier-Stokes equations of motion is carried out with respect to time. This averaging leads to Reynolds Averaged Navier-Stokes equations (RANS). Additional terms with new variables occur in these partial differential equations because of the averaging. Consequently there are suddenly more variables than equations. In order to close the motion equation system in this study, $k-\varepsilon$ turbulence modeling will be used.

2. THE OBJECTIVES OF THIS STUDY:

- To simulate numerically fluid flow in an enclosure using k-epsilon turbulence model
- To generate isotherms and streamlines for different aspect ratios.
- To observe the effect of the aspect ratio along hot and cold walls
- To use the numerical results in making conclusions and making recommendations.

3. LITERATURE REVIEW:

Aydin *et al*. (1999) investigated natural convection in rectangular enclosure heated from one side and cooled from the ceiling.

Betts & Bokhari (2000) found that the partially conducting roof and floor provided locally unstable thermal stratification in the wall jet flows across the diagonal of a tall differentially heated rectangular cavity.

Peng & Davidson (2001) studied turbulent natural convection flow in a confined cavity with two differentially heated side walls.

Bilgen, E. (2002) studied Laminar and turbulent natural convection in enclosures with partial partitions by a numerical method.

A three dimensional rectangular enclosure containing a convectional heater built into one wall and having a window in the same wall study was done by Sigey *et al*. (2004).

Sharma *et al*. (2007) did Conjugate turbulent natural convection and surface radiation in rectangular enclosures heated from below and cooled from other walls.

Braga, & de Lemos, (2009) did study on Turbulent natural convection in a two-dimensional horizontal composite square cavity.

Sigey, J. K. (2012) solved equations governing natural convection in a rectangular enclosure.

Awuor, K.O. (2013) studied performance of three numerical turbulence models in turbulent Convection Fluid Flow in an Enclosure. The non-linear terms in the averaged momentum and energy equations were modeled using the k -ε , k -ω and k -ω - *SST* models to close the governing equations. He found that k -ω - *SST* model performed better than both k - ε and k -ω models in the whole enclosure.

Zhou, W *et al*. (2015) investigated numerically the natural convective flow and heat transfer in a rectangular cavity filled with a heat-generating porous medium

Kumar *et al*. (2016) investigated heat transfer inside a narrow triangular enclosure of aspect ratio 0.3175, between hot base plate and inclined cold wall while others wall being isothermal.

4. GOVERNING EQUATIONS:

The set of governing equations in 2-D rectangular coordinates which are continuity, momentum in X and Y directions ,energy equations turbulent kinetic energy, k and dissipation rate equation, ε are:-

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad \ldots\ldots 1$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=F_x-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \quad \ldots\ldots 2$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=F_Y-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) \quad \ldots\ldots 3$$

$$\rho C_p\left(\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+\Phi \quad \ldots\ldots 4$$

$$\text{Where}\Phi=\mu\left\{2\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2\right]+\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)^2\right\}$$

$$\frac{\partial}{\partial t}(\rho k)+\bar{u}\frac{\partial}{\partial x_i}(\rho k\mu_i)=\frac{\partial}{\partial x_j}\left[\left(\mu+\frac{V_T}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right]+G_K+G_b-\rho\varepsilon-\mathrm{Y_m}+\mathrm{S_k} \quad \ldots\ldots 5$$

$$\frac{\partial}{\partial t}(\rho\varepsilon)+\bar{u}\frac{\partial}{\partial x_i}(\rho\varepsilon\mu_i)=\frac{\partial}{\partial x_j}\left[\left(\mu+\frac{V_T}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial x_j}\right]+C_{1\varepsilon}\frac{\varepsilon}{K}(G_K+C_{3\varepsilon}G_b)-C_{2\varepsilon}\frac{\varepsilon^2}{\mathrm{k}}+\mathrm{S}_\varepsilon \quad \ldots\ldots 6$$

Non – dimensionalizing the governing equations makes the equations simpler and highlights which terms are the most important. The main objective behind non – dimensionalization is to reduce the number of parameters. The set of Equations 1,2,3 and 4 should be solved to obtain the unknowns u, v, p and T. By applying Boussinesq approximation and then introducing dimensionless parameters U, V, θ,τ ,P, X and Y;

$$X=\frac{x}{L}, \qquad Y=\frac{y}{L}, U=\frac{uL}{\alpha_f}, V=\frac{vL}{\alpha_f}, \theta_f=\frac{T_f-T_c}{T_h-T_c}$$

$$., \tau=\frac{\alpha_f t}{L^2}, \text{p}=\frac{L^2 p}{\rho\alpha_f^2} \quad \ldots\ldots 7$$

The set of equation in dimensionless form can be written as:

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \quad \ldots\ldots 8$$

$$\frac{\partial U}{\partial \tau}+U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial X}+Pr\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right) \quad \ldots\ldots 9$$

$$\frac{\partial V}{\partial \tau}+U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial Y}+Pr\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)+Ra.Pr.\theta_f \quad \ldots\ldots 10$$

$$\left(\frac{\partial \theta_f}{\partial \tau}+U\frac{\partial \theta_f}{\partial X}+V\frac{\partial \theta_f}{\partial Y}\right)=k\left(\frac{\partial^2 \theta_f}{\partial X^2}+\frac{\partial^2 \theta_f}{\partial Y^2}\right)+\Phi \quad \ldots\ldots 11$$

where Ra and Pr represents, Rayleigh and Prantdl numbers respectively; and is dimensionless temperature of fluid.

By employing the dimensionless vorticity and dimensionless stream function parameters, the dimensionless form of the governing equations is obtained where the pressure term in the momentum equation is eliminated. With the application of vorticity-streamfunction approach, the following equations are obtained to find the unknown velocities and the temperature values;

$$\frac{\partial \Omega}{\partial \tau}+\frac{\partial U\Omega}{\partial X}+\frac{\partial V\Omega}{\partial Y}=Pr\left(\frac{\partial^2 \Omega}{\partial X^2}+\frac{\partial^2 \Omega}{\partial Y^2}\right)+RaPr\frac{\partial \theta_f}{\partial X} \quad \ldots 12$$

$$\frac{\partial^2 \psi}{\partial X^2}+\frac{\partial^2 \psi}{\partial Y^2}=-\Omega \quad \ldots 13$$

$$\frac{\partial \theta_f}{\partial \tau}+\frac{\partial U\theta_f}{\partial X}+\frac{\partial V\theta_f}{\partial Y}=\frac{\partial^2 \theta_f}{\partial X^2}+\frac{\partial^2 \theta_f}{\partial Y^2} \quad \ldots 14$$

Where

$$Ra=\frac{g\beta(T_h-T_c)L^3}{\upsilon \propto}$$

5. MATHEMATICAL FORMULATION:

Figure 1 shows a schematic diagram of the problem under consideration and the coordinate system. The system to be considered is a two – dimensional rectangular cavity of width W and height H, where the two vertical are kept at different temperatures, T_h (left wall) and T_c (right wall), $T_h > T_c$. Zero heat flow is assumed at the top and bottom walls (adiabatic). The walls are rigid and no – slip conditions are imposed at the boundaries.

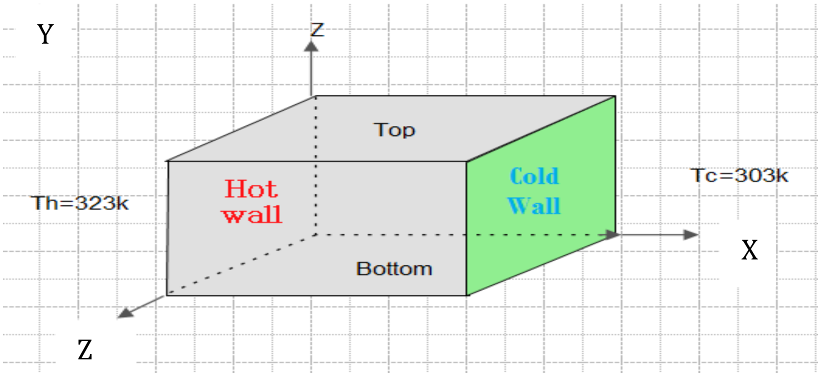


Fig.5.1

Since the energy equation 14 and the vorticity equation 12 are similar to each other, they can be expressed in the form of a single generic equation (Mobedi 1994);

$$\frac{\partial \emptyset}{\partial \tau}+U\frac{\partial \emptyset}{\partial X}+V\frac{\partial \emptyset}{\partial Y}=C\left(\frac{\partial^2 \emptyset}{\partial X^2}+\frac{\partial^2 \emptyset}{\partial Y^2}\right)+f \quad \ldots 15$$

Where $\emptyset$ is a generic dependent variable representing Ω.

6. FINITE DIFFERENCE SOLUTION METHOD FOR PARABOLIC DIFFERENTIAL EQUATIONS:

Equation 15 can be reduced to the following form;

$$\frac{\partial \emptyset}{\partial \tau}=\delta_X^2\emptyset+\delta_Y^2\emptyset+f \quad \ldots 16$$

In equation 16, $\delta_X^2\emptyset$ and $\delta_Y^2\emptyset$ are

$$\delta_X^2\emptyset=C\frac{\partial^2 \emptyset}{\partial X^2}-U\frac{\partial \emptyset}{\partial X} \quad \ldots 17$$

$$\delta_Y^2\emptyset=C\frac{\partial^2 \emptyset}{\partial Y^2}-V\frac{\partial \emptyset}{\partial Y} \quad \ldots 18$$

Term $\delta_X^2\emptyset$ and $\delta_Y^2\emptyset$ refer to diffusion and convection transport in X and Y directions, respectively. For this reason, they can be called as diffusion-convection terms. The parabolic partial differential equations can be solved by various finite difference methods. Those methods are generally classified into three types, namely, explicit, implicit and ADI (Alternating Direction Implicit) methods (Thiault 1985).

Application of the ADI method on Equation 16 for any node (i, j) in Cartesian coordinates when a simple forward difference for the time term is used can be written in two steps as;

$$\frac{\emptyset_{i,j}^{n+1/2}-\emptyset_{i,j}^{n}}{\Delta\tau/2}=\delta_X^2\emptyset_{i,j}^{n+1/2}+\delta_Y^2\emptyset_{i,j}^{n}+f_{i,j}^{n} \quad \ldots 19$$

$$\frac{\emptyset_{i,j}^{n+1}-\emptyset_{i,j}^{n+1/2}}{\Delta\tau/2}=\delta_X^2\emptyset_{i,j}^{n+1/2}+\delta_Y^2\emptyset_{i,j}^{n+1}+f_{i,j}^{n+1/2} \quad \ldots 20$$

Where the Equation 19 is implicit for x-direction and explicit for y-direction and the Equation 20 is implicit for y-direction and explicit for x-direction.

Equation 16 can be arranged as;

$\left(1-\frac{\Delta\tau}{2}\delta_X^2\right)\emptyset_{i,j}^{n+1/2}=\left(1+\frac{\Delta\tau}{2}\delta_Y^2\right)\emptyset_{i,j}^{n}+\frac{\Delta\tau}{2}f_{i,j}^{n}$……………………………………………..….21

Similarly, equation 20 can be arranged as

$\left(1-\frac{\Delta\tau}{2}\delta_Y^2\right)\emptyset_{i,j}^{n+1}=\left(1+\frac{\Delta\tau}{2}\delta_X^2\right)\emptyset_{i,j}^{n}+\frac{\Delta\tau}{2}f_{i,j}^{n+1/2}$…..…………………………………….…..22

As it can be seen, the major advantage of ADI method with respect to a fully implicit method is that the solution of the parabolic differential equation for a time step can be obtained after two steps.

7. RESULTS AND DISCUSSIONS:

The results presented here were obtained by solving the governing equations numerically using Finite Difference Method and together with the boundary conditions give the numerical solutions for variables in $\boldsymbol{\kappa}-\boldsymbol{\epsilon}$ model.

In this study, height is kept constant at 2m while changing the distance between two isothermal walls i.e. the left and right walls which in this case is referred as the aspect ratio. The aspect ratio is varied at a sequence of even numbers (2,4,6, and 8) and results of isotherms, stream lines and contours of velocity magnitudes are recorded at z = 0.5.

Isotherms

Isotherm is a line of equal or constant temperature or is a curve on a graph that connects points of equal temperature.

In figure 7.1a, the highest temperature is 130 K, In 7.1 b, the highest temperature is 75.8 K, 7.1c, the highest temperature is 26.7 K and in 16 d, the highest temperature is 13.1K. The high temperatures are evident on the left side wall. In all cases two circular motion in different directions (one in clockwise and the other one in anticlockwise direction).There is rises up of hot less dense particles which losses its heat with distance as shown by change in color. In between the two isothermal walls there is mixing of air particles which is a region of thermal equilibrium and is a relatively cold region. In 7.1 c and 7.1 d, temperature uniformity is achieved. In conclusion, it is evident that highest temperature decreases with increase in aspect ratio.

Figure 7.1a isotherm of aspect ratio 2

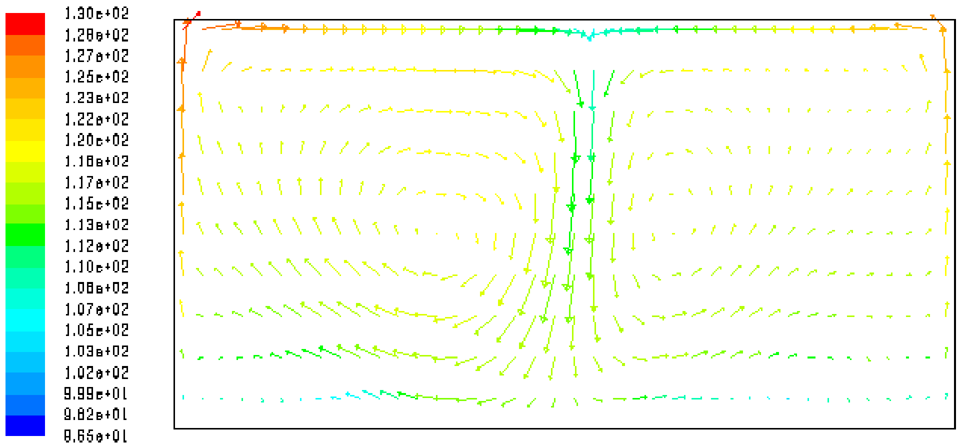


Figure 7.1b isotherm of aspect ratio 4

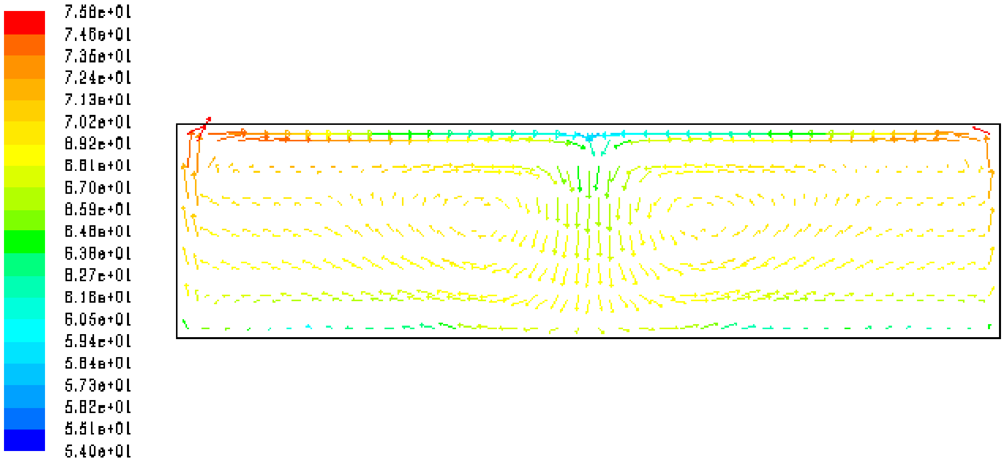


Figure 7.1c isotherm of aspect ratio 6

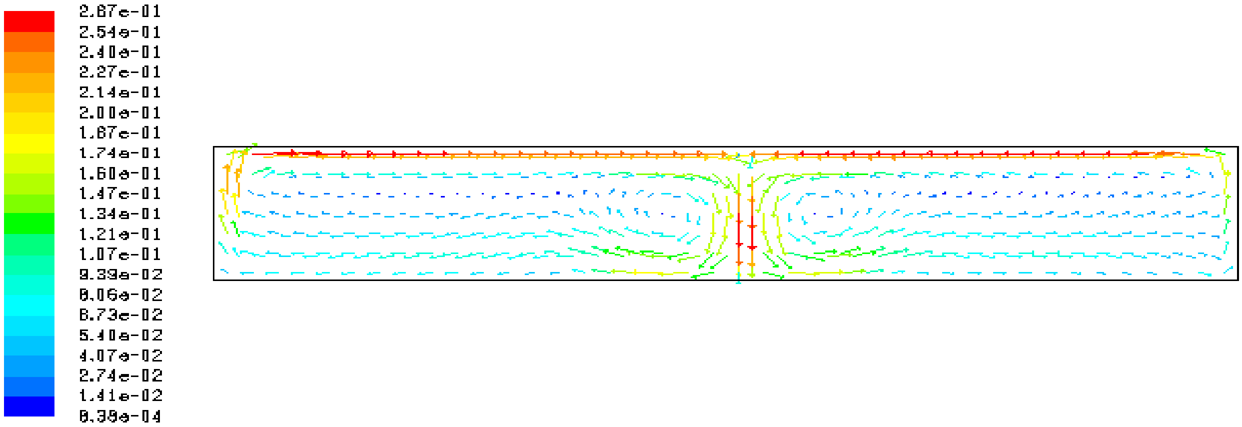
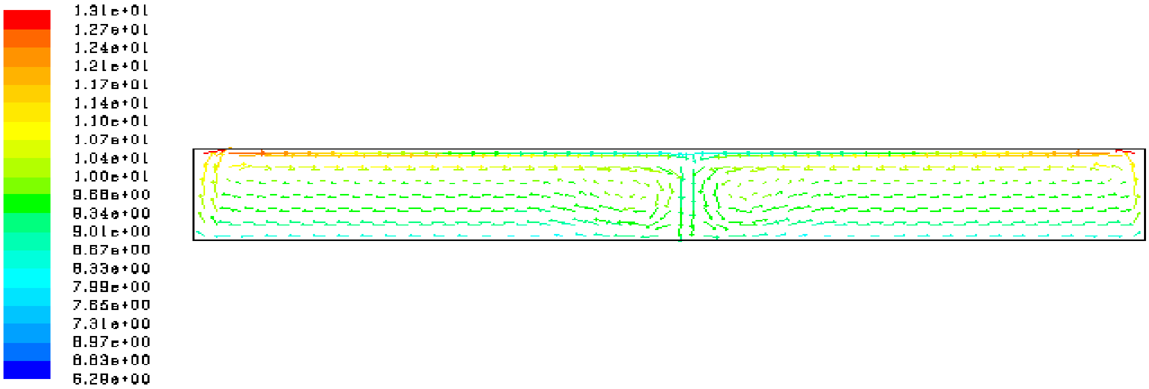


Figure 7.1d isotherm of aspect ratio 8



7.2 Contours of Velocity Magnitudes

In 7.2a the highest velocity of air particles is 0.5m/s, in 6.2b the highest velocity is 0.285 m/s, in 7.2c the highest velocity is 0.246 m/s and in 7.2d the highest velocity is 0.204 m/s. In 6.2a, the highest speed is at the middle – at the mixing region. Vortices are more in 7.2a which become parallel as aspect ratio increases. In 7.2d are parallel than any other set up in this study and at this point is evident that as aspect ratio increases the flow becomes less turbulent.

Figure 7.2a Contours of velocity magnitude (m/s) of aspect ratio 2

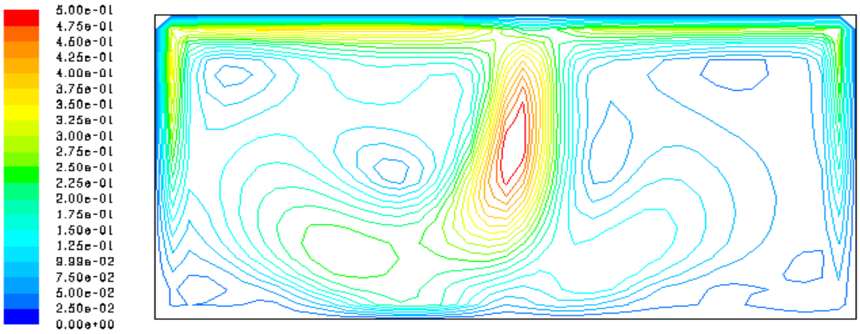


Figure 7.2b Contours of velocity magnitude (m/s) of aspect ratio 4

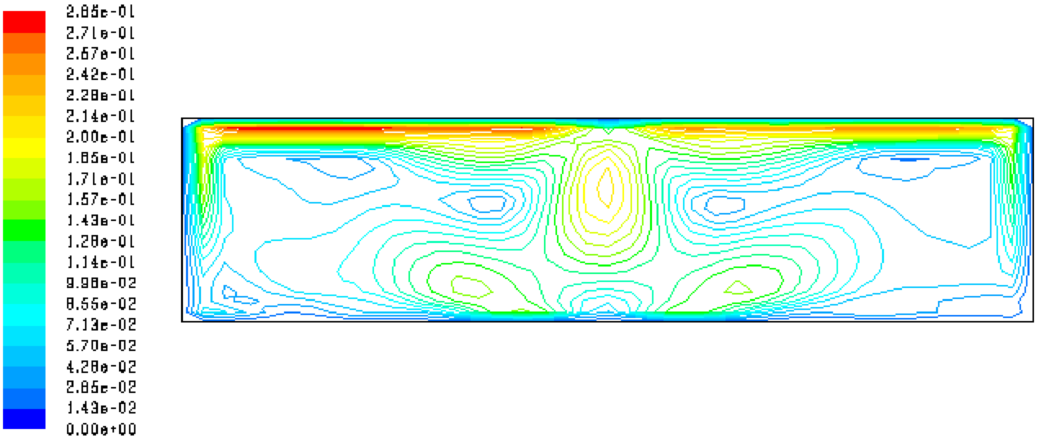


Figure 7.2c Contours of velocity magnitude (m/s) of aspect ratio 6

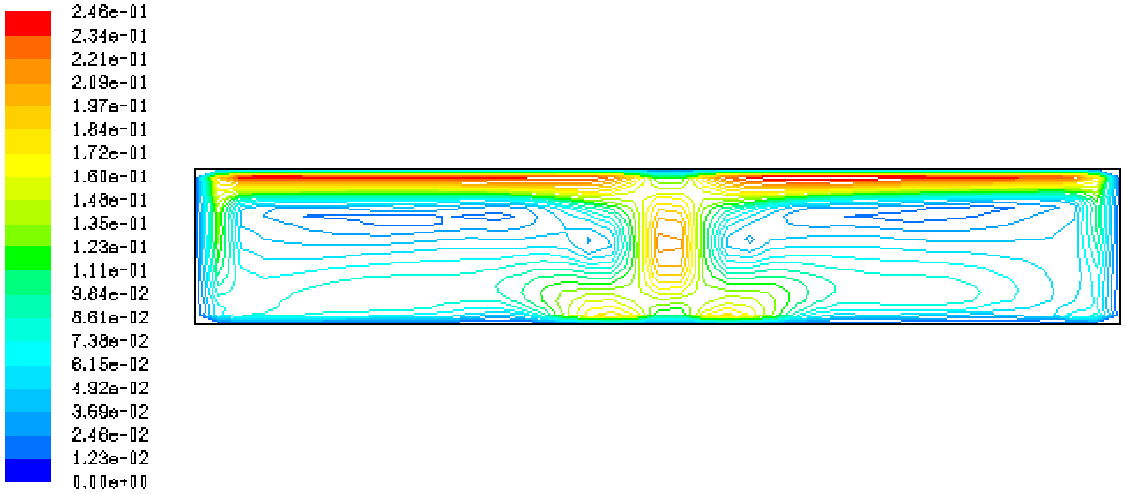
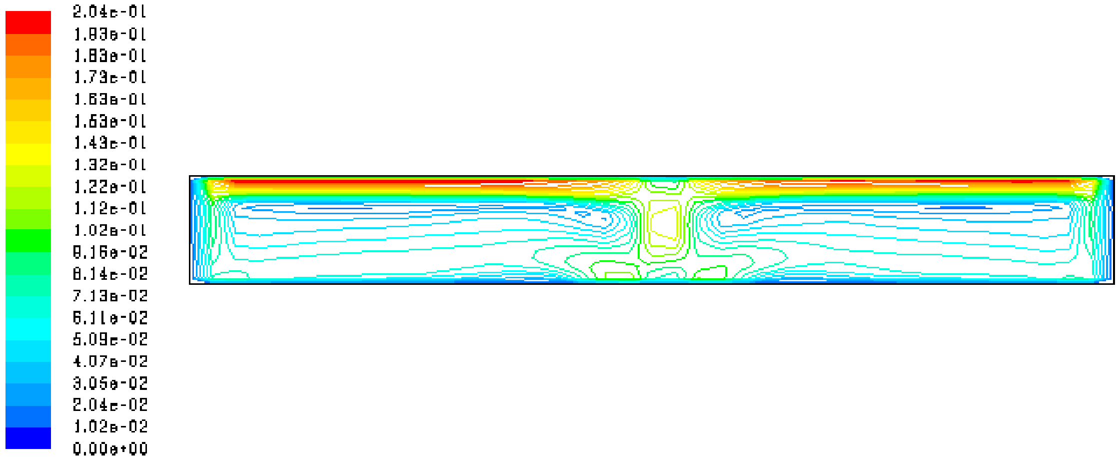


Figure 7.2d Contours of velocity magnitude (m/s) of aspect ratio 8



7.3 Streamline Distribution

A streamline is an imaginary line in a fluid such that the tangent at any point indicates the direction of the velocity of a particle of the fluid at that point.

The highest value indicated here is that of aspect ratio 2 which is 0.293Kg/s followed by that of aspect ratio 4 which is 0.254 Kg/s. This value reduces as aspect ratio increases as depicted by that of aspect ratio 6 which is 0.238 Kg/s and the lowest which is 0.178 Kg/s as shown by that of aspect ratio 8. In 7.3a, the vortices are big in size at the two centers and they assume a circular path which deforms as distance increases from their centers. In 7.3b, radius of centre circle reduces which as well decreases as the aspect ratio increases to 8 as seen in 7.3c. In 6.3d the two centre cell deforms and takes an oval shape. The vortices become parallel as aspect ratio increases.

Figure 7.3a Contours of streamlines (kg/s) of aspect ratio 2

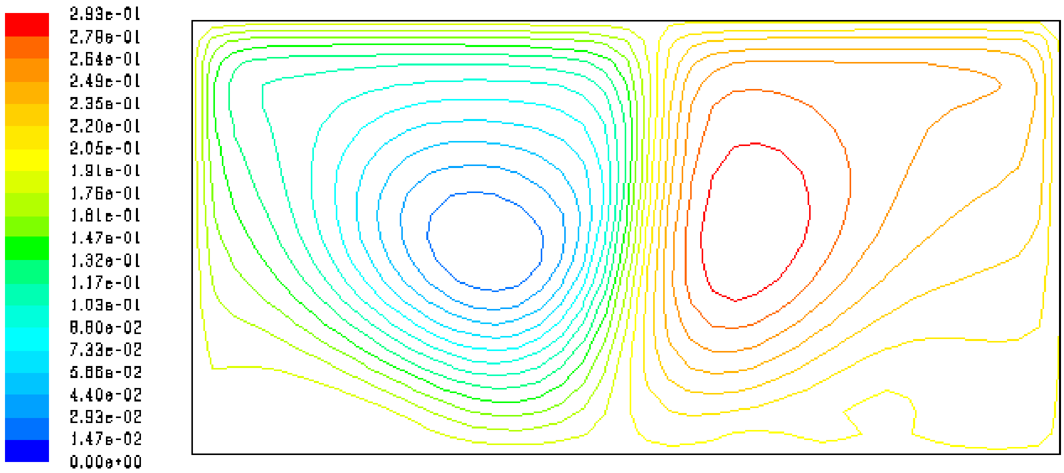


Figure 7.3bContours of streamlines (kg/s) of aspect ratio 4

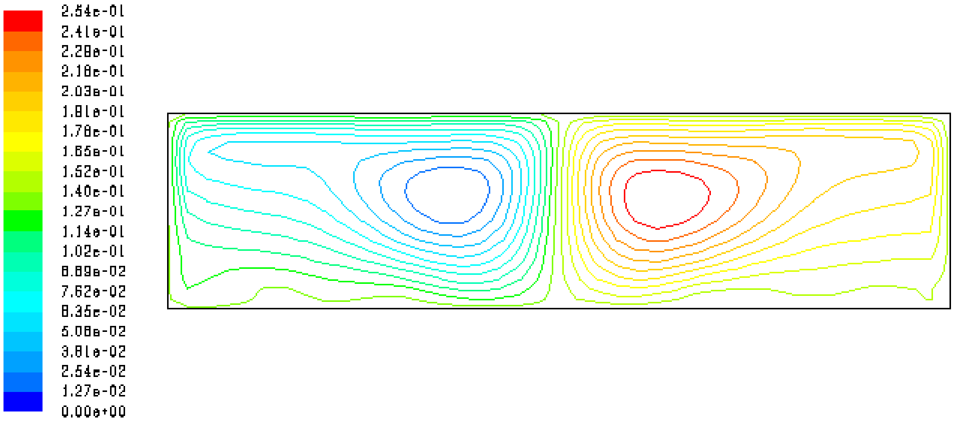


Figure 7.3c Contours of streamlines (kg/s) of aspect ratio 6

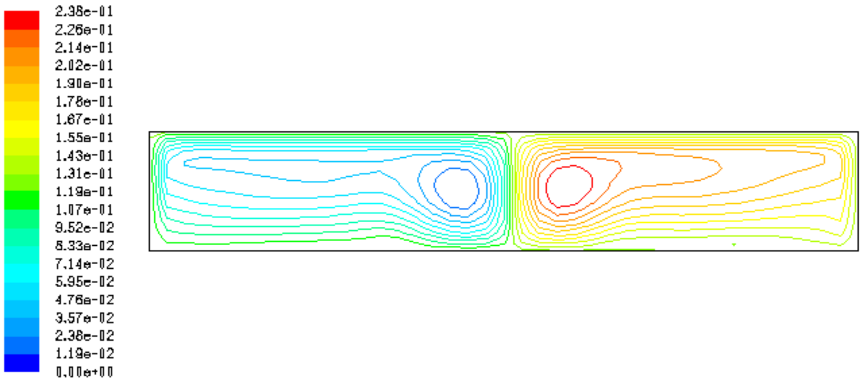
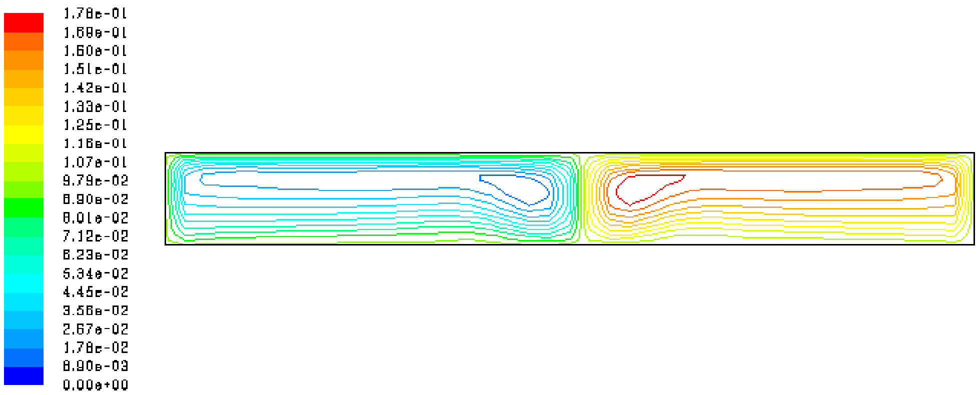


Figure 7.3d Contours of streamlines (kg/s) of aspect ratio 8



7. CONCLUSION:

The objective of the study was to generate isotherms and streamlines for different aspect ratios and observe the effect of the aspect ratio along hot and cold walls in a enclosure. The heat transfer was by convection. The solutions were obtained for aspect ratio of 2, 4, 6, and 8. The geometry considered is a rectangular enclosure in form of a room with left wall at higher temperature than the right side wall as shown in Fig 1.

The Boussinesq approximations were used, allowing the conservation equations to be simplified. The governing equations with boundary conditions were discretized using a three-point central and forward difference approximation.

The results show that as the aspect ratio increased speed decreased and vortices became more parallel thus decreasing turbulence. So, the aspect ratio has a significant effect in fluid flow and temperature field in horizontal enclosures heated from the side. This helps in keeping of some items at the stated temperature as well as in brine exclusion

8. RECOMMENDATIONS:

Further investigations are recommended for

- Enclosures with a three-dimensional configuration.

- The effect of the changing Rayleigh number and aspect ratio.
- Same configuration but using $k - \omega\, SST$ model, RNG $k - \varepsilon$ model and Realizable $k - \varepsilon$
- Varying the characteristics of the fluid contained in the enclosure.

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