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Some Significant Sets and Some Significant Functions

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Abstract: *The purpose of this research paper is to explore how wonderful of significant sets and significant functions which have different properties from usual set and usual function encountered in pure Mathematics. These significances can be obtained by means of properties of compact set, equivalence set and metric space together with properties of sequential continuity in a metric space. Moreover, we also discuss some illustrative example as well as counter-examples where necessary. Some expository result of significance of sets and functions will be obtained by means of exploring properties of sets and functions and comparing these properties among them.*

Key Words: *Compact Set, Equivalence Set (Class), Sequential Continuity, Metric Space.*

1. INTRODUCTION:

There are two main significances of set and function. Among them these are two kinds of significances of set about compact set and equivalence set which have finite intersection property, (generalized as Mazur property) and equivalence class property which can be partitioned into disjoint subsets of a set. Likewise, significances of function can be seen by exploring properties of continuity and sequential continuity which have domains being metric spaces. By altering other topological spaces in places of metric spaces, we can see different nature of the function.

2. SOME KIND OF SIGNIFICANT SETS:

Some kind of significant sets are first kind of significance of sets, equivalence set definitions, second kind of significance of sets, compact set definition and finite intersection property.

2.1 First Kind of Significance of Sets

T For any set A and B, A∩ B may be empty or non-empty depending on A and B.
If A ≠B, it may be A⊂B or B⊂A or (both A⊄B and B⊄A).
For A⊂B, A∩ B = A.
For B ⊂ A, A∩ B = B.
For A⊄B and B⊄A, $A \cap B \neq \emptyset$ or not
A∩ B = A =B may be empty or not since it depends on A = B = Ø or not.

If A∩ B then A∩ B = A =B may be empty or not since it depends on A = B = Ø or not. If there is an ordering between A and B, then we can see that A∩ B = A = B or A∩ B = Ø. Such sets can be found in the study of equivalence sets. Equivalence sets are significant among usual sets.

2.1.1 Definition

Let A ≠ Ø.

Let R ⊂ A×A be a relation.
For x, y∈ A, we say **x and y are related** if (x,y) ∈R, denoting as x ≤ y.

(i) R is said to be **reflexive** if (x, x) ∈ R for all x ∈ A.
(ii) R is said to be **symmetric** if (x, y) ∈ R implies (y, x) ∈R for any x, y ∈ R.
(iii) R is said to be **transitive** if (x, y) ∈ R and (y, z) ∈ R implies (x, z) ∈ R for any x, y, z ∈ A.
(iv) R is said to be **an equivalence relation** if R holds (i), (ii), and (iii) [1, 2].

2.1.2 Example

Let x be a fixed integer in Z where n ≠0.
For p, q ∈ Z, we define p ≤ q,
if $(n|(p-q))\ or\ p \equiv q\ mod\ n$.

For any p ∈ Z, p – p = 0 and hence $(n|(p-p))$.
Then, p ≤ p for any p ∈ Z.
Hence the relation is reflexive on Z.
Again, for any p, q ∈ Z, Let p≤q. Then, $(n|(p-q))$.

So, $(n|-(p-q))\ or\ (n|(q-p))$.
Then , q ≤ p.
Hence, the relation is symmetric on Z.
For p, q, r ∈ Z, let p ≤ q and q≤ r.
Then, $n|p-q\ and\ n|q-r$.
So, $\left(n\middle|\left((p-q)+(q-r)\right)\right)$
i.e. $(n|(p-r))$
Then, p ≤ r.
Hence, the relation is transitive.
Finally, we can say that the relation is an equivalence relation.

2.1.3 Example

Let x, y ∈R.
We define x ≤ y if x < y in R(x is less than y)

Then the relation is not reflexive since x< x does not hold for any real number x. Likewise, the relation is not symmetric. So, the relation is not an equivalence relation although it is transitive.

2.1.4 Example

Let x, y ∈ R.
We define: x ≤ y if x is less than or equal to y.
Since x = x for any x ∈ R, x ≤ x holds.
So, the relation is reflexive.
For x, y ∈ R such that x is less than or equal y.
If x = y then y = x and hence y ≤ x but if x< y then y> x is false.
So, the relation is not reflexive.
Although, the relation is transitive, it is not an equivalence relation on R.

2.1.5 Example

Let x, y ∈ R.
We define x ≤ y, if x = y in R.
Then x = x for any x ∈ R.
So, the relation is reflexive.
For x, y ∈ R, x ≤ y holds.
Then x = y.
So, y = x and hence y ≤ x.
Thus, the relation is reflexive.
For x, y, z ∈ R, x ≤ y and y ≤ z holds.
Then x = y and y = z.
So, x = z.

Then, x ≤ z.
Thus, the relation is transitive. Finally we can say that the relation is an equivalence relation.

2.2 Equivalence Set Definitions

Let, A ≠Ø and R be an equivalence relation on A.
Let x, ∈ A.
We define the set $E_x$or Cl(x) or $\tilde{x}$ by $\tilde{x} = \{y \in A|(x, y) \in R\}$ or
$= \{y \in A|y \le x\}$
Then $\tilde{x}$ (E_x or Cl (x)) is called an equivalence set or equivalence class containing x [2].

2.2.1 Theorem

Let A ≠Ø.
Let ≤ be an equivalence relation on A [2].
Then, (i) $E_x \neq Ø$ for x∈ A.
(ii) x ≤ y implies $E_x = E_y$.
(iii) $E_x = E_y$ implies x ≤y.
(iv) For x is not related to y, $E_x \cap E_y = ϕ$, its converse is also true.
(v) $A = \bigcup_{x\in A} E_x$.

Proof: For any x ∈ A, x≤ x since ≤ is reflexive.
So, x ∈ E_x.
Then, $E_x \neq \emptyset$ for any x ϵ A.
Hence, (i) holds.
For (ii), let x≤ y for x, y ϵA.

By (i), y ∈ E_y .
Since ≤ is symmetric, y ≤x.
By definition of $E_x, y \in E_x$.

So, y∈ E_y implies that y ∈ E_x .
Then, $E_y \subset E_x$.
Again, let z∈ E_x.
By definition of E_x, z ≤ x.
Since, x ≤ y and ≤ is transitive,
z ≤ x, x ≤ y implies that z ≤ y.

So, z ∈ E_y.
Hence, z ∈ E_x implies that z ∈ E_y.
Then, $E_x \subset E_y$.
Finally, we have $E_x = E_y$.
So,(ii) holds.
For (iii), let $E_x = E_y$.
Then p ∈ E_x implies that p ∈ E_y.

So, p ≤ x and p≤ y.
Since ≤ is reflexive, x ≤ p for p ≤y.
So, x ≤ p and p ≤ y.
Then, x ≤ y since ≤ is transitive.
So, (iii) holds.
For (iv), let x be not related to y for x, y∈ A.
We will prove: $E_x \cap E_y = \emptyset$.
Suppose contrary that : $E_x \cap E_y \neq \emptyset$.
Then : $\exists z \in A$: $z \in E_x \cap E_y$.

So, $z \in E_x$ and $z \in E_y$.
Then, z ≤ x and z ≤ y.

Since $\leq$ is reflexive, $x \leq z$.

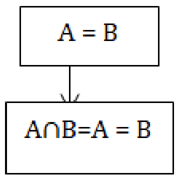
So, $x \leq z$ and $z \leq y$.
Since $\leq$ is transitive, $x \leq y$.
It is a contradiction to the fact that x is not related to y.
So, it is false that $E_x \cap E_y \neq \emptyset$.
Then, $E_x \cap E_y = \emptyset$.
For the converse statement, let $E_x \cap E_y = \emptyset$.
We will prove that x is not related to y in A.
Suppose contrary that $x \leq y$ in A.
By (ii), $E_x = E_y$ and hence $E_x \cap E_y = E_x$.
By (i), $E_x \neq \emptyset$ for any $x \in A$.
So, $E_x \cap E_y \neq \emptyset$.
It is a contradiction to $E_x \cap E_y = \emptyset$.
So, it is false that $x \leq y$.
Then x is not related to y.
So, (iv) holds.

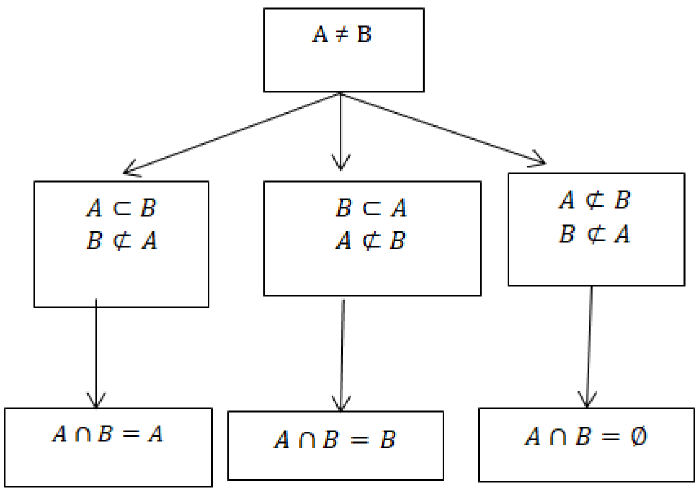
Remark

By above theorem, equivalence sets are significant among usual sets encountered in in pure mathematics.
For any usual set A and B,
$A \cap B = A$ or B if $A = B$ only.
i.e. if $A = B$ then $A \cap B = A = B$.
But for any equivalence sets E_x *and* E_y, either $E_x \neq E_y$ or not.
For distinct elements $x, y \in A$, we have
Either, $x \leq y$ or not.
For $x \leq y$, $E_x = E_y$ by Theorem 1.2.1 (iii).
Then, $E_x \cap E_y = E_x = E_y \neq \emptyset$.
For x and y which are not related, $E_x \cap E_y = \emptyset$.
Indeed, E_x *and* E_y are different according to their construction.

$$E_x = \{p \in A : p \leq x\} \text{ and } E_y = \{q \in A | q \leq y\}$$

For usual sets A and B, either A = B or not.





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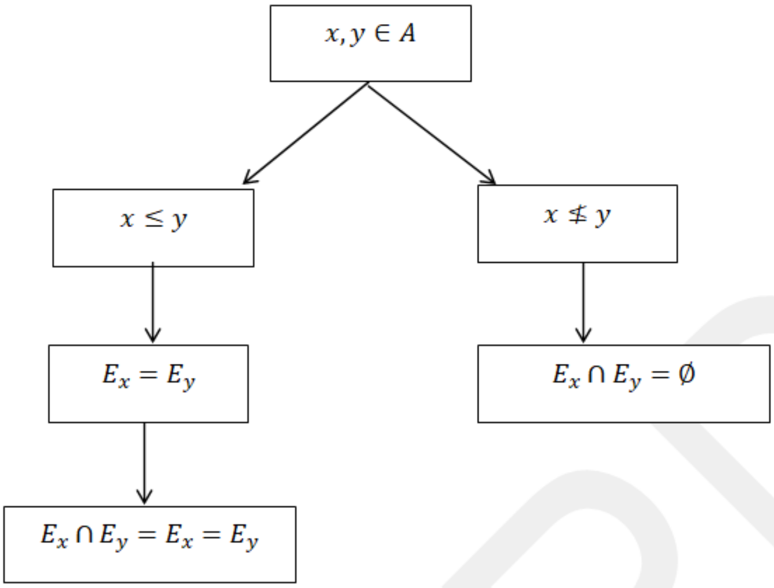
Hence, there are three cases for usual sets.
For equivalence sets E_x and E_y, there are two cases only.



Note that there is no special cases: $E_x \cap E_y = E_x$ or $E_x \cap E_y = E_y$.
i.e., there are neither $E_x \cap E_y = E_x$ nor $E_x \cap E_y = E_y$.

2.3 Second Kind of Significance of Sets

It is about intersection of compact sets.
For usual sets A_i, $\bigcap_{i=1}^{n} A_i$ may not be empty but $\bigcap_{i=1}^{\infty} A_i = \emptyset$.

Consider, let A_1= { 1, 2, 3}, A_2 ={2, 3, 4} and A_3 = {1, 4}.
$A_1 \cap A_2 = \{2, 3\} \neq \emptyset$.
$A_1 \cap A_3 = \{1\} \neq \emptyset$.
$A_2 \cap A_3 = \{4\} \neq \emptyset$.
But, $A_1 \cap A_2 \cap A_3 \neq \emptyset$.

For several numbers of usual sets, their intersections are forced to empty since there is no common element which belongs to each set. This condition is not true for compact sets because they possess the finite intersection property: there is no empty intersection of infinite number or compact sets if there is no empty intersection of finite number of these compact sets. This property was extended as Mazur property if we consider totally bounded sets in place of compact sets.

2.4 Compact Set Definition

Let X be a topological space.
Suppose A ⊂ X.
We say A is a compact set if open covering of the set A has a finite open sub covering [5].
i.e, If A ⊂ $\bigcup_{\propto} O_{\propto}$ for each open set $O_{\propto}$, there is a finite number of open set $O_{\propto_1}, O_{\propto_2}, \ . \ . \ . \ , O_{\propto_n}$
Such that $A \subset \bigcup_{i=1}^{n} o_{\alpha_i}$

2.4.1 Theorem

Compact subset of a metric space is closed [4,5]. Proof: See [4].

2.4.2 Theorem

Closed subset of a compact set is closed [4,5]. Proof: See [4].

2.4.3 Theorem [4, 5]

If $\{K_n\}$ is a sequence of non-empty compact sets such that

$K_{n+1} \subset K_n$ for n= 1, 2, 3, then, $\bigcap_{n=1}^{\infty} K_n$ is non-empty. Proof: See [4].

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2.4.4 Theorem

Let k be a positive integer.
Let I_n consists of all points $x = (x_1, . . . , x_k)$
Such that,

$$a_{n,j} \le x_j \le b_{n,j} (1 \le j \le k; n = 1,2,3, \ . \ . \ .) \ and \ I_{n,j} = [a_{nj}, b_{nj}].$$

Then $\{I_n\}$ is a sequence of k-cell and every k-cell is compact [5]. Proof: See [5].

2.5 Finite Intersection Property

It is a property especial for compact sets. If a finite number of intersection of compact sets in non-empty, then its infinite number of intersection of these compact set is non-empty [5].

2.5.1 Theorem (Finite Intersection Property) (FIP)

Let X be a topological space.

Let $\{K_\propto\}$ be a collection of compact subsets of X and the intersection of finite sub-collection of $\{K_\propto\}$ is non-empty, then $\bigcap_\propto K_\propto$ is non-empty [5].

Poof:

Fix a member K_1 of $\{K_\propto\}$ and $G_\propto = K_\propto^c$.
Assume that no point of K_1 belongs to every$K_\propto$.
The set $G_\propto$ is open since $K_\propto$ is closed.
So, the sets $G_\propto$ form an open covering of K_1 because $x \in K_1$ implies that $x \notin K_\propto$ for each $\propto$ (or) $x \in K_1$ implies $x \in K_\propto^c = G_\propto$ for each $\propto$.
So, $K_1 \subset G_\propto$ and hence, $K_1 \subset \bigcup_\propto G_\propto$.
Since K_1 is compact, there are finitely many $\propto_1, \propto_2, \ . \ . \ . \ , \propto_n$
Such that,

$$K_1 \subset G_{\propto_1} \cup G_{\propto_2} \cup . \ . \ . \cup G_{\propto_n}$$
(or) $K_1 \subset K_{\propto_1}^c \cup K_{\propto_2}^c \cup . \ . \ . \cup K_{\propto_n}^c$
(or) $K_1 \subset (K_{\propto_1} \cap K_{\propto_2} \cap . \ . \ . \cap K_{\propto_n})^c$
So, $K_1 \cap (K_{\propto_1} \cap K_{\propto_2} \cap . \ . \ . \cap K_{\propto_n}) = \emptyset$

It is a contradiction to our hypothesis.
So, it is false that $K_1 \subset K_\propto^c$ for each $\propto$.
So, $\bigcap_\propto K_\propto = \emptyset$ is false.
Thus, $\bigcap_\propto K_\propto \neq \emptyset$.

2.5.2 Remark

In classical theory, $\bigcap_\propto K_\propto = \emptyset$ for a finite intersection of non-empty and non-compact sets [3].
Let $K_n = \left(0, \frac{1}{n}\right), n \in \mathbb{Z}^+$.
For a finite number m,

$$\bigcap_{n=1}^{m} K_n = (0,1) \cap \left(0, \frac{1}{2}\right) \cap \left(0, \frac{1}{3}\right) \cap \ . \ . \ . \cap (0, \frac{1}{m}) = \left(0, \frac{1}{m}\right) \neq \emptyset$$

For $m \to \infty$, $\bigcap_{n=1}^{\infty} K_n = \lim_{m\to\infty} \left(0, \frac{1}{m}\right) = (0,0) = \emptyset$.

Hence, FIP does not hold for non-empty and non-compact sets.

3. SOME KIND OF SIGNIFICANT FUNCTIONS:

Some kind of significant functions are first kind of significant functions and second kind of significant function.

3.1 First Kind of Significant Functions

First kind of significant functions are definition, example and theorem.

3.1.1 Definition

Let (X,d_X) be a metric space with metric $d_X$and (Y, d_Y) be also a metric space with metric d_Y [4].
Suppose f: X→ Y
x⟼f(x) be a function.

Let a ∈X.
We say f is continuous at x = a if for any ε >0, there is a positive number δ >0 such that
d_X (x, a) < δ implies that d_Y (f(x), f (a)) < ε.
If f is continuous at arbitrary point a ∈X, f is continuous in X.
Now, δ > 0 depends on ε and a if f is continuous at a.
i.e.,$\delta = \delta(\varepsilon, a)$.

3.1.2 Example

Let $f: \mathbb{R} \to \mathbb{R}$

$$x \longmapsto f(x) = x^2, a \in \mathbb{R}.$$

$$\begin{aligned} d(f(x), f(a)) &= |f(x) - f(a)| \\ &= |x^2 - a^2| \\ &= |x - a||x + a| \\ &\leq |x - a|(|x| + |a|) \end{aligned}$$

For, $|x| = |x - a + a|$,
$|x| \leq |x - a| + |a|$
For, $|x - a| \leq \delta$,
$|x| \leq \delta + |a|$
So, $|f(x) - f(a)| \leq \delta(\delta + |a|)$
$= \delta^2 + \delta|a|$
$\leq \delta^2 + \delta(1 + |a|)$.

Let ε > 0.
In order to get $|f(x) - f(a)| < \varepsilon$,
$\delta^2 + \delta(1 + |a|) \quad \varepsilon$.

Choose δ > 0 so that $\delta^2 + \delta(1 + |a|) < \varepsilon$.
Then, $|x - a| \leq \delta$ implies $|f(x) - f(a)| < \varepsilon$.
So, f is continuous of x = a.

Note: Since δ be chosen so that $\delta^2 + \delta(1 + |a|) < \varepsilon$, δ depends on both a and ε. If a and ε change, δ will change .Such continuity is called point-wise continuity.

3.1.3 Example

Let f: $\mathbb{R} \longrightarrow \mathbb{R}$

$$x \longmapsto f(x) = 2x + 1.$$

Let a∈ $\mathbb{R}$.
Then, $|f(x) - f(a)| = |2(x - a)| = 2|x - a|$.
Let ε > 0.
In order to get $|f(x) - f(a)| < \varepsilon$, $2|x - a| < \varepsilon$ (or) $|x - a| < \frac{\varepsilon}{2}$.
Choose δ > 0 so that $0 < \delta < \frac{\varepsilon}{2}$.
Then, $|x - a| < \delta$ implies that $|f(x) - f(a)| = 2|x - a| < 2\delta < 2\frac{\varepsilon}{2} = \varepsilon$.
So, f is continuous at x = a.

Now, δ depends on ε only, but not on the point a we considered. Such kind of continuity is called uniform continuity.

3.1.4 Definition

Let (x,d_X) and (Y, d_Y) be metric spaces with metrics d_X and d_Y respectively [4,5].
Let f: X → Y
$x \longmapsto f(x)$ be a function.
Suppose a be arbitrary in X.

We say f is uniformly continuous in X if for any ε > 0, there is a positive number δ> 0 depending on ε only but not for the point a if d_X(x, a) < δ implies that $d_Y(f(x), f(a)) < \varepsilon$.

3.1.5 Theorem

Le (X, d_X) be a compact metric space and (Y, d_Y) a metric space [5].
Let f: X → Y
$x \longmapsto f(x)$ be continuous in X.
Then f is uniformly continuous in X.

Proof:

Let $\varepsilon > 0$.
Since f is continuous, we can associate to each point p ∈X, there is a positive number $\phi(p) > 0$.
Such that
q ∈ X, d_X(p, q) < $\phi(p)$ implies d_Y (f(p), f(a)) < $\frac{\varepsilon}{2}$.
Let J(p) = $\left\{q \in X \middle| d_X(p,q) < \frac{1}{2}\phi(p)\right\}$.

Since p ∈ J (p), the collection of all sets J(p) is an open cover of X, and since X is compact, there is a finite set of point $p_1, p_2, \ . \ . \ ., p_n$ in X such that $X \subset J(p_1) \cup J(p_2) \cup \ . \ . \ . \cup J(p_n)$. We put

$$\delta = \frac{1}{2}\min\{\phi(p_1), \phi(p_2), \ . \ . \ ., \phi(p_n)\}.$$

Then $\delta > 0$.
Now, let q and p be points of X such that d_X(p,q) < δ.
Then there is an integer m (1≤m≤n) such that p ∈ $J(p_m)$.
Hence $d_X(p,p_m) < \frac{1}{2}\phi(p_m)$ and we have

$$d_X(p,q) \le d_X(p,q) + d_X(p,p_m) < \delta + \frac{1}{2}\phi(p_m) \le \phi(p_m).$$

Therefore,

$$d_Y(f(p), f(q)) \le d_Y\big(f(p), f(p_m)\big) + d_Y\big(f(p), f(p_m)\big) < \varepsilon.$$

Then f is uniformly continuous.

Remark

By above theorem, compactness of domain of a continuous function makes uniformly continuous function. In example 2.1.3, the domain of this quadratic function is ℝ and ℝ is not compact. So, the quadratic function is just point-wise continuous but not uniformly continuous. Hence, compactness of domain of a continuous function plays an important role to signify the continuous function.

3.2 Second Kind of Significant Function

This kind of significance is the special property of a metric space in a metric space, sequential continuity becomes continuity. Generally, every continuous function is sequentially continuous but its converse may not be true except a metric space.

3.2.1 Definition

Let (X, τ_X) and (Y, τ_Y) be topological space together with respective topologies τ_X and τ_Y [5].
Suppose f : X → Y
$x \longmapsto f(x)$.

(i) We say f is **continuous** in X if V is open in Y implies $f^{-1}(V)$ is open in X.
(ii) We say f is **sequentially continuous** at a point x in X if x_n converges to x in X implies $f(x_n)$ converges to f(x) in Y.

3.2.2 Theorem

Let X and Y be topological spaces.
Then f is continuous at x ∈ X implies that f is sequentially continuous at x in X [4].

Proof:

Let f : X → Y be continuous at x in X.
We will prove: f is sequentially continuous at x in X.
Let $x_n \to x$.
We need to prove: $f(x_n) \to f(x)$ as $n \to \infty$.

Let V be an open set such that $f(x) \in V \subset Y$.
Then, $x \in f^{-1}(V)$.
Let $\varepsilon > 0$.
Since $x_n \to x$, there exists an integer N such that
$n \geq N$ implies that $x_n \in f^{-1}(V)$.
So, $x \in f^{-1}(V)$ implies that $x_n \in f^{-1}(V)$ for all $n \geq N$.
Then, $f(x) \in V$ implies that $f(x_n) \in V$ for all $n \geq N$.
Hence, $f(x_n) \to f(x)$.
So, f is sequentially continuous at x in X.
Thus, continuity implies sequential continuity.

3.2.3 Theorem

In a metric space X, sequential continuity implies continuity [4].

Proof:

Take any x in X.
Let V be an open set containing f(x).
Then, $f(x) \in V$ and hence $x \in f^{-1}(V)$.
Let $x_n \to x$ imply $f(x_n) \to f(x)$.
Suppose contrary that f is not continuous at x.
Then, $x \in f^{-1}(V)$ but $f^{-1}(V)$ is not open in X.
So, x is not an interior point of $f^{-1}(V)$.
For all r > 0, $N_r(x) \not\subset f^{-1}(V)$.
So, there is $p \in X$: $p \in N_r(x)$ and $p \notin f^{-1}(V)$.
Since $x_n \to x$, there exists $N \in \mathbb{Z}^+$: $n \geq N$ implies that $x_n \in N_r(x)$.
Then, $x_n \notin f^{-1}(V)$ for some $n \geq N$ since p is one of $x_n's$.
So, $f(x_n) \notin V$ for some $n \geq N$.
Then, $f(x_n)$ does not converge to $f(x)$ in Y
Hence, it contradicts our hypothesis.
So, it is false that f is not continuous at x in X
Hence, f is continuous in X.

Note: In above theorems, the domain of being a metric space plays an important role to signify sequential continuity into continuity. Thus, sequential continuity coincides continuity in a metric space. If the topological space is not a metric space, this coincidence is false. See the following: example.

3.2.4 Example

Let us consider $\mathbb{R}$ and $\tau = \{A^c \subset \mathbb{R} | A \text{ is countable}\} \cup \{\phi\}$ [3,4].
Clearly, $\phi \in \tau$. Since, ϕ is finite, $\phi^c \in \tau$.i.e., $\mathbb{R} \in \tau$.
Let A and B be countable.
Then $A^c, B^c \in \tau$.
Since $A \cup B$ is countable, $(A \cup B)^c \in \tau$
i.e., $A^c \cap B^c \in \tau$
For countable set A_i, $A_i^c \in \tau$.
Then $\bigcap_i A_i$ is countable and hence.
$(\bigcap_i A_i)^c \in \tau$.
i.e, $\bigcup_i A_i^c \in \tau$.
Hence, τ is a topology on $\mathbb{R}$.
So, $(\mathbb{R}, \tau)$ is a topological space.
Moreover, $(\mathbb{R}, \mathcal{U})$ is a topological space where

$$\mathcal{U} = \{A \subset \mathbb{R} | \forall x \in A, \exists r > 0: (x - r, x + r) \subset A\}.$$

Consider $f: (\mathbb{R}, \tau) \to (\mathbb{R}, \mathcal{U})$
$x \longmapsto f(x) = x$

For $(0,1) \in \mathcal{U}, f^{-1}\big((0,1)\big) = (0,1)$.

Have $(0,1)^c = (-\infty, 0] \cup [1, \infty)$ is not countable.

So, $(0,1) \notin \tau$.
i.e. $f^{-1}(0,1) \notin \tau$.

So, $f^{-1}(0,1)$ is not open with respect to τ.
This, f is not continuous.
But f is sequentially continuous.

Let $x_n \to x$ in $(\mathbb{R}, \tau)$.
Then, $f(x_n) = x_n$ and f(x) = x.
So, $f(x_n) \to f(x)$ in $(\mathbb{R}, \mathcal{U})$.

Then f is sequentially continuous. Hence, it establishes the failure of coincidence of continuity and sequential continuity if we change a topological space in place of a metric space. This is the also a significance of identify function which is continuous with respect to the usual topology $\mathcal{U}$ but not true for other topological space $(\mathbb{R}, \tau)$ different from a metric space.

4. CONCLUSION:

We have presented how wonderful that significant of sets and functions which have different result of usual sets and functions encountered in pure mathematics. Significant of metric space and identity function have also been described in this paper. More significant can be found by exploring properties of sets and functions and comparing among them. They will be expository results.

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