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ON THE SETS OF INTEGRAL SOLUTIONS TO THE CUBIC EQUATION WITH FOUR UNKNOWNS $x^3+y^3+(x+y)(x-y)^2=16zw^2$

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Abstract: *The homogeneous cubic equation with four unknowns represented by the Diophantine equation $x^3+y^3+(x+y)(x-y)^2=16zw^2$ is analyzed for its patterns of non-zero distinct integral solutions.*

Key Words: *Homogeneous cubic equation, cubic with four unknowns, integral solutions.*

1. INTRODUCTION:

The cubic diophantine equations are rich in variety and offer an unlimited field for research [1-2]. For an extensive review of various problems, one may refer [3-22]. This paper concerns with another interesting cubic diophantine equation with four unknowns $x^3+y^3+(x+y)(x-y)^2=16zw^2$ for determining its infinitely many non-zero integral solutions.

2. METHOD OF ANALYSIS:

The homogeneous cubic equation with four unknowns to be solved for its distinct non-zero integral solution is

$$x^3+y^3+(x+y)(x-y)^2=16zw^2 \tag{1}$$

Introduction of the linear transformations

$$x=u+v,\ y=u-v,\ z=u \tag{2}$$

in (1) leads to

$$u^2+7v^2=8w^2 \tag{3}$$

Different methods of obtaining the patterns of integer solutions to (1) are illustrated below:

PATTERN 1:

We can write (3) in the form of ratio as

$$\frac{u+w}{7(w+v)}=\frac{(w-v)}{u-w}=\frac{\alpha}{\beta},\ \beta\neq 0$$

The above equation is equivalent to the double equations

$$\beta u-7\alpha v+(\beta-7\alpha)w=0 \quad \text{and}$$

$$\alpha u+\beta v-(\alpha+\beta)w=0$$

Applying the method of cross multiplication, we get

$$\left.\begin{aligned} u&=7\alpha^2+14\alpha\beta-\beta^2 \\ v&=-7\alpha^2+2\alpha\beta+\beta^2 \end{aligned}\right\} \tag{4}$$

$$w(\alpha,\beta)=\beta^2+7\alpha^2 \tag{5}$$

Using (4) in (2), we have

$$\left.\begin{aligned} x(\alpha,\beta)&=16\alpha\beta\\ y(\alpha,\beta)&=14\alpha^2+12\alpha\beta-2\beta^2\\ z(\alpha,\beta)&=7\alpha^2+14\alpha\beta-\beta^2\end{aligned}\right\} \tag{6}$$

Thus, (5) and (6) give a set of integer solutions for (1).

PATTERN 2:

We can write (3) in the form of ratio as

$$\frac{u+w}{(w+v)}=\frac{7(w-v)}{u-w}=\frac{\alpha}{\beta},\ \beta\neq 0$$

The above equation is equivalent to the double equations

$$\beta u-\alpha v+(\beta-\alpha)w=0 \quad \text{and}$$

$$\alpha u+7\beta v-(\alpha+7\beta)w=0$$

Applying the method of cross multiplication, we get

$$\left.\begin{aligned} u&=\alpha^2+14\alpha\beta-7\beta^2\\ v&=-\alpha^2+2\alpha\beta+7\beta^2\end{aligned}\right\} \tag{7}$$

$$w(\alpha,\beta)=7\beta^2+\alpha^2 \tag{8}$$

Using (7) in (2), we have

$$\left.\begin{aligned} x(\alpha,\beta)&=16\alpha\beta\\ y(\alpha,\beta)&=2\alpha^2+12\alpha\beta-14\beta^2\\ z(\alpha,\beta)&=\alpha^2+14\alpha\beta-7\beta^2\end{aligned}\right\} \tag{9}$$

Thus, (8) and (9) represents the integer solutions to (1).

PATTERN 3:

Let

$$w=a^2+7b^2 \tag{10}$$

where a and b are non-zero integers.

Write 8 as

$$8=\left(1+i\sqrt{7}\right)\left(1-i\sqrt{7}\right) \tag{11}$$

Using (10), (11) in (3) and applying the method of factorization, define

$$\left(u+i\sqrt{7}v\right)=\left(1+i\sqrt{7}\right)\left(a+i\sqrt{7}b\right)^2 \tag{12}$$

from which we have

$$\left.\begin{aligned} u&=a^2-14ab-7b^2\\ v&=a^2+2ab-7b^2\end{aligned}\right\} \tag{13}$$

Using (7) and (2), the values of x, y and z are given by

$$\left.\begin{aligned} x &= x(a,b) = 2a^2 - 12ab - 14b^2 \\ y &= y(a,b) = -16ab \\ z &= z(a,b) = a^2 - 14ab - 7b^2 \end{aligned}\right\}$$

(14)

Thus (10) and (14) represent the non-zero integer solutions to (1).

PATTERN 4:

Write 8 as

$$8 = \frac{(5 + i\sqrt{7})(5 - i\sqrt{7})}{4} \tag{15}$$

Using (10), (15) in (3) and applying the method of factorization, define

$$\left(u + i\sqrt{7}v\right) = \left(\frac{5 + i\sqrt{7}}{2}\right)\left(a + i\sqrt{7}b\right)^2 \tag{16}$$

from which we have

$$\left.\begin{aligned} u &= \frac{1}{2}\left(5a^2 - 14ab - 35b^2\right) \\ v &= \frac{1}{2}\left(a^2 + 10ab - 7b^2\right) \end{aligned}\right\} \tag{17}$$

Since our interest is on finding integer solutions, replacing a by 2A, b by 2B in (10) and (17), the corresponding integer solutions to (1) are given by

$$\left.\begin{aligned} x &= x(A,B) = 12A^2 - 8AB - 84B^2 \\ y &= y(A,B) = 8A^2 - 48AB - 56B^2 \\ z &= z(A,B) = 10A^2 - 28AB - 70B^2 \\ w &= w(A,B) = 4A^2 + 28B^2 \end{aligned}\right\}$$

(18)

Thus (18) represent the non-zero integer solutions to (1).

PATTERN 5:

Write 8 as

$$8 = \frac{(11 + i\sqrt{7})(11 - i\sqrt{7})}{16} \tag{19}$$

Using (10), (19) in (3) and applying the method of factorization, define

$$\left(u + i\sqrt{7}v\right) = \left(\frac{11 + i\sqrt{7}}{4}\right)\left(a + i\sqrt{7}b\right)^2 \tag{20}$$

from which we have

$$\left.\begin{aligned} u &= \frac{1}{4}\left(11a^2 - 14ab - 77b^2\right) \\ v &= \frac{1}{4}\left(a^2 + 22ab - 7b^2\right) \end{aligned}\right\} \tag{21}$$

Since our interest is on finding integer solutions, replacing a by 2A, b by 2B in (10) and (21), the corresponding integer solutions to (1) are given by

$$\left.\begin{aligned} x &= x(A,B) = 12A^2 + 8AB - 84B^2 \\ y &= y(A,B) = 10A^2 - 36AB - 70B^2 \\ z &= z(A,B) = 11A^2 - 14AB - 77B^2 \\ w &= w(A,B) = 4A^2 + 28B^2 \end{aligned}\right\} \tag{22}$$

Thus (22) represent the non-zero integer solutions to (1).

PATTERN 6:

Write (3) as

$$u^2+7v^2=8w^2*1 \tag{23}$$

write 1 as

$$1=\left(\frac{(3+i\sqrt{7})(3-i\sqrt{7})}{16}\right) \tag{24}$$

Using (10), (11), (24) in (23) and applying the method of factorization, define

$$(u+i\sqrt{7}v)=(1+i\sqrt{7})(a+i\sqrt{7}b)^2\left(\frac{3+i\sqrt{7}}{4}\right) \tag{25}$$

from which we have

$$\left.\begin{aligned} u&=(-a^2-14ab+7b^2)\\ v&=(a^2-2ab-7b^2)\end{aligned}\right\} \tag{26}$$

Using (26) and (2), the values of x, y and z are given by

$$\left.\begin{aligned} x&=x(a,b)=-16ab\\ y&=y(a,b)=-2a^2-12ab+14b^2\\ z&=z(a,b)=-a^2-14ab+7b^2\end{aligned}\right\} \tag{27}$$

Thus (10) and (27) represent the non-zero integer solutions to (1).

PATTERN 7:

Assume 1 as

$$1=\left(\frac{(1+i3\sqrt{7})(1-i3\sqrt{7})}{64}\right) \tag{28}$$

Using (10), (11),(28) in (23) and applying the method of factorization, define

$$(u+i\sqrt{7}v)=(1+i\sqrt{7})(a+i\sqrt{7}b)^2\left(\frac{1+i3\sqrt{7}}{8}\right) \tag{29}$$

from which we have

$$\left.\begin{aligned} u&=\frac{1}{2}(-5a^2-14ab+35b^2)\\ v&=\frac{1}{2}(a^2-10ab-7b^2)\end{aligned}\right\} \tag{30}$$

Since our interest is on finding integer solutions, replacing a by $2A$, b by $2B$ in (10) and (30), the corresponding integer solutions to (1) are given by

$$\left.\begin{aligned} x&=x(A,B)=(-8A^2-48AB+56B^2)\\ y&=y(A,B)=(-12A^2-8AB+84B^2)\\ z&=z(A,B)=(-10A^2-28AB+70B^2)\\ w&=w(A,B)=(4A^2+28B^2)\end{aligned}\right\} \tag{31}$$

Thus (31) represent the non-zero integer solutions to (1).

PATTERN 8:

Assume 1 as

$$1=\left(\frac{(3+i4\sqrt{7})(3-i4\sqrt{7})}{121}\right) \tag{32}$$

Using (10), (11),(32) in (23) and applying the method of factorization, define

$$\left(u+i\sqrt{7}v\right)=\left(1+i\sqrt{7}\right)\left(a+i\sqrt{7}b\right)^2\left(\frac{3+i4\sqrt{7}}{11}\right) \tag{33}$$

from which we have

$$\left.\begin{aligned} u&=\frac{1}{11}\left(-25a^2-98ab+175b^2\right)\\ v&=\frac{1}{11}\left(7a^2-50ab-49b^2\right)\end{aligned}\right\} \tag{34}$$

Since our interest is on finding integer solutions, replacing a by 11A, b by 11B in (10) and (34), the corresponding integer solutions to (1) are given by

$$\left.\begin{aligned} x&=x(A,B)=\left(-198A^2-1628AB+1386B^2\right)\\ y&=y(A,B)=\left(-352A^2-528AB+2464B^2\right)\\ z&=z(A,B)=\left(-275A^2-1078AB+1925B^2\right)\\ w&=w(A,B)=(121A^2+847B^2)\end{aligned}\right\} \tag{35}$$

Thus (35) represent the non-zero integer solutions to (1).

PATTERN 9:

Assume 1 as

$$1=\left(\frac{\left(1+i48\sqrt{7}\right)\left(1-i48\sqrt{7}\right)}{127^2}\right) \tag{36}$$

Using (10), (11),(36) in (23) and applying the method of factorization, define

$$\left(u+i\sqrt{7}v\right)=\left(1+i\sqrt{7}\right)\left(a+i\sqrt{7}b\right)^2\left(\frac{1+i48\sqrt{7}}{127}\right) \tag{37}$$

from which we have

$$\left.\begin{aligned} u&=\frac{1}{127}\left(-335\,a^2-686\,ab+2345\,b^2\right)\\ v&=\frac{1}{127}\left(49\,a^2-670\,ab-343\,b^2\right)\end{aligned}\right\} \tag{38}$$

Since our interest is on finding integer solutions, replacing a by 127A, b by 127B in (10) and (38), the corresponding integer solutions to (1) are given by

$$\left.\begin{aligned} x&=x(A,B)=\left(-36322A^2-172212AB+254254B^2\right)\\ y&=y(A,B)=\left(-48768A^2-2032AB+341376B^2\right)\\ z&=z(A,B)=\left(-42545A^2-87122AB+297815B^2\right)\\ w&=w(A,B)=(16129A^2+112903B^2)\end{aligned}\right\} \tag{39}$$

Thus (39) represent the non-zero integer solutions to (1).

PATTERN 10:

Assume 1 as

$$1=\left(\frac{\left(3+i4\sqrt{7}\right)\left(3-i4\sqrt{7}\right)}{121}\right) \tag{40}$$

Using (10), (9),(34) in (17) and applying the method of factorization, define

$$\left(u+i\sqrt{7}v\right)=\left(\frac{5+i\sqrt{7}}{2}\right)\left(a+i\sqrt{7}b\right)^2\left(\frac{3+i4\sqrt{7}}{11}\right) \tag{41}$$

from which we have

$$\left.\begin{aligned} u &= \frac{1}{22}\left(-13a^2 - 322ab + 91b^2\right) \\ v &= \frac{1}{22}\left(23a^2 - 26ab - 161b^2\right) \end{aligned}\right\} \quad (42)$$

Since our interest is on finding integer solutions, replacing a by 22A, b by 22B in (10) and (42), the corresponding integer solutions to (1) are given by

$$\left.\begin{aligned} x &= x(A,B) = \left(220A^2 - 7656AB - 1540B^2\right) \\ y &= y(A,B) = \left(-792A^2 - 6512AB + 5544B^2\right) \\ z &= z(A,B) = \left(-286A^2 - 7084AB + 2002B^2\right) \\ w &= w(A,B) = (484A^2 + 3388B^2) \end{aligned}\right\}$$

(43)

Thus (43) represent the non-zero integer solutions to (1).

PATTERN 11:

Assume 1 as

$$1 = \left(\frac{\left(1 + i48\sqrt{7}\right)\left(1 - i48\sqrt{7}\right)}{127^2}\right) \quad (44)$$

Using (10), (15),(44) in (23) and applying the method of factorization, define

$$\left(u + i\sqrt{7}v\right) = \left(\frac{5 + i\sqrt{7}}{2}\right)\left(a + i\sqrt{7}b\right)^2\left(\frac{1 + i48\sqrt{7}}{127}\right) \quad (45)$$

from which we have

$$\left.\begin{aligned} u &= \frac{1}{254}\left(-331a^2 - 3374ab + 2317b^2\right) \\ v &= \frac{1}{254}\left(241a^2 - 662ab - 1687b^2\right) \end{aligned}\right\}$$

(46)

Since our interest is on finding integer solutions, replacing a by 254A, b by 254B in (10) and (46), the corresponding integer solutions to (1) are given by

$$\left.\begin{aligned} x &= x(A,B) = \left(-22860A^2 - 1025144AB + 160020B^2\right) \\ y &= y(A,B) = \left(-145288A^2 - 688848AB + 1017016B^2\right) \\ z &= z(A,B) = \left(-84074A^2 - 856996AB + 588518B^2\right) \\ w &= w(A,B) = (64516A^2 + 451612B^2) \end{aligned}\right\} \quad (47)$$

Thus (47) represent the non-zero integer solutions to (1).

PATTERN 12:

Assume 1 as

$$1 = \left(\frac{\left(3 + i\sqrt{7}\right)\left(3 - i\sqrt{7}\right)}{16}\right) \quad (48)$$

Using (10), (19),(48) in (23) and applying the method of factorization, define

$$\left(u + i\sqrt{7}v\right) = \left(\frac{11 + i\sqrt{7}}{4}\right)\left(a + i\sqrt{7}b\right)^2\left(\frac{3 + i\sqrt{7}}{4}\right) \quad (49)$$

from which we have

$$\left.\begin{aligned} u &= \frac{1}{8}\left(13\,a^2 - 98\,ab - 91\,b^2\right) \\ v &= \frac{1}{8}\left(7\,a^2 + 26\,ab - 49\,b^2\right) \end{aligned}\right\} \quad (50)$$

Since our interest is on finding integer solutions, replacing a by 8A, b by 8B in (10) and (50), the corresponding integer solutions to (1) are given by

$$\left.\begin{aligned} x &= x(A,B) = \left(160A^2 - 576AB - 1120B^2\right) \\ y &= y(A,B) = \left(48A^2 - 992AB - 336B^2\right) \\ z &= z(A,B) = \left(104A^2 - 784AB - 728B^2\right) \\ w &= w(A,B) = (64A^2 + 448B^2) \end{aligned}\right\} \quad (51)$$

Thus (51) represent the non-zero integer solutions to (1).

PATTERN 13:

Assume 1 as

$$1 = \left(\frac{\left(1 + i3\sqrt{7}\right)\left(1 - i3\sqrt{7}\right)}{64}\right) \quad (52)$$

Using (10), (19),(52) in (23) and applying the method of factorization, define

$$\left(u + i\sqrt{7}v\right) = \left(\frac{11 + i\sqrt{7}}{4}\right)\left(a + i\sqrt{7}b\right)^2\left(\frac{1 + i3\sqrt{7}}{8}\right) \quad (53)$$

from which we have

$$\left.\begin{aligned} u &= \frac{1}{16}\left(-5\,a^2 - 238\,ab + 35\,b^2\right) \\ v &= \frac{1}{16}\left(17\,a^2 - 10\,ab - 119\,b^2\right) \end{aligned}\right\}$$

(54)

Since our interest is on finding integer solutions, replacing a by 16A, b by 16B in (10) and (54), the corresponding integer solutions to (1) are given by

$$\left.\begin{aligned} x &= x(A,B) = \left(192A^2 - 3968AB - 1344B^2\right) \\ y &= y(A,B) = \left(-352A^2 - 3648AB + 2464B^2\right) \\ z &= z(A,B) = \left(-80A^2 - 3808AB + 560B^2\right) \\ w &= w(A,B) = (256A^2 + 1792B^2) \end{aligned}\right\} \quad (55)$$

Thus (55) represent the non-zero integer solutions to (1).

PATTERN 14:

Assume 1 as

$$1 = \left(\frac{\left(3 + i4\sqrt{7}\right)\left(3 - i4\sqrt{7}\right)}{121}\right) \quad (56)$$

Using (10), (19),(56) in (23) and applying the method of factorization, define

$$\left(u + i\sqrt{7}v\right) = \left(\frac{11 + i\sqrt{7}}{4}\right)\left(a + i\sqrt{7}b\right)^2\left(\frac{3 + i4\sqrt{7}}{11}\right) \quad (57)$$

from which we have

$$\left.\begin{aligned} u &= \frac{1}{44}\left(5a^2 - 658\,ab - 35\,b^2\right) \\ v &= \frac{1}{44}\left(47\,a^2 + 10\,ab - 329\,b^2\right) \end{aligned}\right\} \quad (58)$$

Since our interest is on finding integer solutions, replacing a by 44A, b by 44B in (10) and (58), the corresponding integer solutions to (1) are given by

$$\left.\begin{aligned} x &= x(A,B) = \left(2288A^2 - 28512AB - 16016B^2\right) \\ y &= y(A,B) = \left(-1848A^2 - 29392AB + 12936B^2\right) \\ z &= z(A,B) = \left(220A^2 - 28952AB - 1540B^2\right) \\ w &= w(A,B) = (1936A^2 + 13552B^2) \end{aligned}\right\} \quad (59)$$

Thus (59) represent the non-zero integer solutions to (1).

3. CONCLUSION:

In this paper, we have made an attempt to determine different patterns of non-zero distinct integer solutions to the homogeneous cubic equation with four unknowns given by $x^3 + y^3 + (x+y)(x-y)^2 = 16zw^2$. As the cubic equations are rich in variety, one may search for other forms of cubic equations with multi-variables to obtain their corresponding solutions.

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