INTERNATIONAL JOURNAL FOR INNOVATIVE RESEARCH IN MULTIDISCIPLINARY FIELD ISSN: 2455-0620 Volume - 6, Issue - 12, Dec – 2020
Monthly, Peer-Reviewed, Refereed, Indexed Journal with IC Value: 86.87 Impact Factor: 6.719
Received Date: 14/12/2020 Acceptance Date: 26/12/2020 Publication Date: 31/12/2020

Prime Labelling For Duplication Of Graph Elements In A Complete Bipartite Graph $K_{m,n} - e$.

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Abstract: *Let us consider a graph G. Here the vertices are connected to distinct integers. Let v=ab denotes an edge, now if the labels given to a and b are relatively prime then the graph G is said to have prime labelling. If a graph admits prime labelling then it is said be a prime graph. Here we are going to check the prime labelling for $K_{m,n}$-e ,where e is an edge. We are also going to discuss the prime labelling for duplication in $K_{m,n}$-e.*

Keywords: *Prime labelling, Complete bipartite graph, Duplication.*

1. INTRODUCTION:

A graph is nothing but a mathematical representation of networks by using points and lines. Here we say the points as vertices and lines as edges. A graph is usually represented as G (V, E), where V represent the vertex set and E represent the edge set.

Here we have considered a bipartite graph. A graph G is called as a bipartite graph if its vertices can be divided into two disjoint sets such that an edge exists between a vertex of one set and the vertex of another set (there exists no edge between the vertices of the same set).

Let us consider a graph G with vertex set V=V(G) and edge set E=E(G). Let N(u) denote the set of vertices adjacent to a vertex u of G. Refer to Bondy and Murthy for any notations and terminology [1].

Roger Entringer introduced the notion of prime labelling and Tout discussed that in a paper [9]. If the greatest common divisor of two integers a and b is 1 then they are said to be relatively prime. Prime graph has been studied by many researchers." A path P_n on n vertices is a prime graph"- this was proved by Fu.H [4]. "The cycle C_n on n vertices is a prime graph" – this was proved by Deretsky et al [3]. "All trees have prime labelling "– this was conjectured by Roger Entringer around 1980, which hasn't settled yet.

"The graphs obtained by identifying any two vertices duplicating arbitrary vertex and switching of any vertex in cycle C_n admit prime labelling"- this was proved by S.K. Vaidhya and K.K. Kanmani in [10]. Prime labelling for some helm related graphs was proved by Meena and Vaithilingam in [6].

Here we will prove that $K_{m,n}$ -e is a prime graph. We are also going to prove that the graph obtained by duplicating it by a vertex and by an edge are all prime graphs in $K_{m,n}$-e.

2. BASIC DEFINITIONS:

2.1 Definition

A graph is nothing but a mathematical representation of networks by using points and lines. Here we say the points as vertices and lines as edges. A graph is usually represented as G (V, E), where V represent the vertex set and E represent the edge set.

2.2 Definition

Let us consider a graph G. Here the vertices are connected to distinct integers. Let v = ab, denote an edge, now if the labels given to a and b are relatively prime then the graph G is said to have prime labelling. If a graph admits prime labelling then it is said be a prime graph.

2.3 Definition

A graph G is called as a bipartite graph if its vertices can be divided into two disjoint sets such that an edge exists between a vertex of one set and the vertex of another set (there exists no edge between the vertices of the same set).

2.4 Definition

If a vertex v of a graph G is duplicated by adding a new vertex v', we obtain a new graph G' such that $N(v') = N(v)$. If the vertices adjacent to v in G are also adjacent to v' then v' is said to be the duplication of v.

2.5 Definition

If the vertex v_k is duplicated by a new edge $e = v_k' v_k''$ in a graph G ,we obtain a new graph G' such that $N(v_k') = \{v_k, v_k''\}$ and $N(v_k'') = \{v_k, v_k'\}$.

2.6 Definition

If an edge e =uv is duplicated by a new vertex w in G ,we obtain a new graph G' such that $N(w)=\{u,v\}$.

2.7 Definition

If an edge e =uv of a graph G is duplicated by adding an edge $e' = u'v'$, we obtain a new graph G' ,such that $N(u') = N(u) \cup \{v'\} - \{v\}$ and $N(v') = N(v) \cup \{u'\} - \{u\}$.

2.8 Definition

Let G be a graph and let $G_1, G_2, \ldots G_n$,$n \geq 2$ be the n copies of that fixed graph G. If we add an edge between G_i andG_{i+1} for i=1,2,…,n-1, we obtain a graph ,which is called as the path union of G.

2.9 Definition

In a graph G ,if every pair of distinct vertices are connected by a unique edge then it is called as a complete graph.

2.10 Definition

In a bipartite graph if every vertex of the first set is connected to every vertex of the second set then it is called as a complete bipartite graph.

3. PRIME LABELLING FOR COMPLETE BIPARTITE GRAPH:

Here we will prove that the graph $K_{m,n}$ -e and the graph obtained by duplicating $K_{m,n}$-e by a vertex and by an edge are all prime graphs .

3.1 Theorem

The graph $K_{m,n}$-e is a prime graph if and only if $n \leq 3$.

Proof:

Let $K_{m,n}$-e be a graph G where $e = v_i v_{i+3}$,i =3.

Then $|V(G)|=n$ and $|E(G)| = \{v_1 v_i, i=2,4,..\} \cup \{v_2 v_i, i=1,3,..\} \cup \ldots \cup \{v_i v_n, i=1,3,\ldots\}$

Define a labelling $f:V(G) \rightarrow \{1,2,\ldots,n\}$ as follows.

$f(v_i)=i$,$1 \leq i \leq n$.

Then f admits prime labelling.

Hence the graph G is a prime graph.

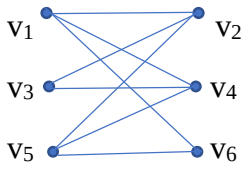

Figure 1

In fig.1, Prime labelling of $K_{m,n}$-e graph is given.

3.2 Theorem

The graph obtained by duplicating a vertex v_2 of $K_{m,n}$-e is a prime graph if and only if $n \leq 2$.

Proof:

On duplicating a vertex v_m in $K_{m,n}$-e ,where $e=v_iv_{i+2}$,i=2 we obtain a graph G.

Then |V(G)|=n+1. Let the new vertex be v_2'. The edge set is

$|E(G)|= \{v_1v_i, i=3,4\} U \{v_2v_i, i=3,4\} U \ldots U \{v_i v_n, i=1,2\}$.

Define a labelling $f:V(G) \rightarrow \{1,2,\ldots,n+1\}$ as follows.

$f(v_i)=i$, $1 \leq i \leq n$.

$f(v_2') =n+1$.

Then f admits prime labelling.

Hence the graph G is a prime graph.

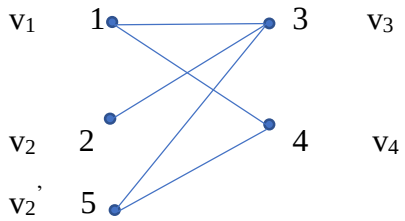

Figure 2

In fig.2, Prime labelling for duplication of v_2 in $K_{m,n}$-e graph is given.

3.3 Theorem

The graph obtained by duplication of a vertex v_3 in $K_{m,n}$-e is a prime graph if and only if n≤3.

Proof:

On duplicating a vertex v_3 in $K_{m,n}$-e ,where $e = v_iv_{i+3}$,i =3 we obtain a new graph G .

Then $|V(G)|=n+1$ and $|E(G)|= \{v_1v_i, i=2,4,..\} U \{v_2v_i, i=1,3,..\} U \ldots U \{v_i v_n, i=1,3,\ldots\}$

Define a labelling $f: V(G) \rightarrow \{1,2,\ldots,n+1\}$ as follows.

$f(v_i)=i$,$1 \leq i \leq n$.

$f(v_3') =n+1$.

Then f admits prime labelling.

Hence the graph G is a prime graph.

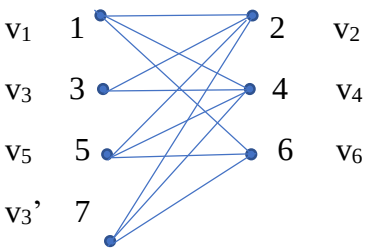

Figure 3

In fig.3, Prime labelling for duplication of v_3 in $K_{m,n}$-e graph is given.

3.4 Theorem

The graph obtained by duplication of an edge in $K_{m,n}$-e is a prime graph if and only if n≤3.

Proof:

On duplicating an edge in $K_{m,n}$-e ,where e = v_iv_{i+3} ,i =3 we obtain a new graph G .

Then |V(G)|=n+2 and |E(G)|= {v_1v_i ,i=2,4,..}U {v_2v_i , i=1,3,..}U…U{$v_i v_n$, i=1,3,…}

Define a labelling f: V(G) →{1,2,…,n+2} as follows.

$f(v_i)$=i ,1 ≤i ≤n,

$f(v_3')$ =n+1,

$f(v_3'')$ =n+2.

Then f admits prime labelling.

Hence the graph G is a prime graph.

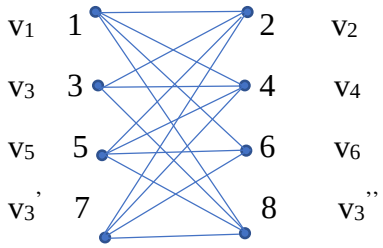

Figure 4

In fig.4, Prime labelling for duplication of v_3 by an edge in $K_{m,n}$-e graph is given.

3.5 Theorem

The graph obtained by duplication of an edge in $K_{m,n}$-e is a prime graph if and only if n≤2.

Proof:

On duplicating a vertex v_m in $K_{m,n}$-e ,where e=v_iv_{i+2},i=2 we obtain a graph G.

Then |V(G)|=n+2.. The edge set is

|E(G)|= {v_1v_i ,i=3,4}U {v_2v_i , i=3,4}U…U{$v_i v_n$, i=1,2}.

Define a labelling f:V(G) →{1,2,…,n+1} as follows.

$f(v_i)$=i, 1 ≤ i ≤ n.

$f(v_2')$ =n+1.

$f(v_2'')$ =n+2.

Then f admits prime labelling.

Hence the graph G is a prime graph.

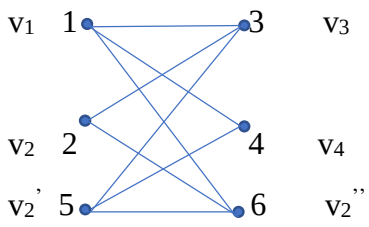

Figure 5

In fig.5, Prime labelling for duplication of v_2 by an edge in $K_{m,n}$-e graph is given.

4. CONCLUSION:

Here, first we have given the definitions of bipartite graph and prime labelling. After that we have proved prime labelling for bipartite graphs. Finally we have proved some results on prime labelling for duplication of graph elements in $K_{m,n}$-e. Hence the prime labelling for duplication of graph elements in complete bipartite graph $K_{m,n}$-e has been proved.

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