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A new method of solution to the underwater acoustic wave guides and it's applications

Dr. John Daniel,
D.Sc. degree researcher, 225/25, Kottai-01, Karaikudi Road, Aranthangi, Pudukottai-614616, Tamil Nadu, India.
Email : johndaniel.india@gmail.com

Abstract: *In this article, a simpler and widely applicable analytical method of solution is presented to shallow water acoustic wave guide problems. The method of coordinate axis transformation and separation of variables are proved to be applicable to solve waveguide problems with variable boundaries and medium. Applications of the method to solve waveguide problems in acoustic, radio and optical frequencies and more general models of waveguides are also explained. The solution by perturbation methods and the method of separation of variables presented earlier does not express the frequency spectrum of the acoustic signal in the explicit manner in addition to narrow domain of applications of the methods. Therefore, a simple and widely applicable method of solution is presented in this article to such wave guide problems which express the frequency spectrum of the acoustic signal very explicitly. Analytical solution to one, two and three dimensional acoustic waveguides are presented.*

Key Words: *Waveguides with surface waves, Perturbation methods, Coordinate transformation and Method of separation of variables.*

1. INTRODUCTION: A.H. Nayfeh developed perturbation methods to study sound propagation in acoustic waveguides with wavy surfaces [1 2]. Many models of oceanic wave guide had been built and many theories were developed to study the propagation of sound waves in shallow water oceanic wave guides. G.V. Anand and M.K. George [3] and V. Sundaravadivel [4] had studied the propagation of sound waves in simple oceanic wave guides with surface waves using analytical methods. They used perturbation methods developed by A.H.Nayfeh [1 2] which are applicable only to wave guides with low frequency and smaller amplitude surface waves. Moreover, perturbation method of solution generates singularity in the solution and complicates the determination of solution at points close to the singularity. Therefore, later on John Daniel [5 6 7 8 9 10 11 12] developed a simpler method of solution using surface wave phase modulation method and the methods of separation of variables which is applicable for any value of surface wave frequency and amplitude and without any singularity in the solution. In this article, a much simpler analytical solution based on the method of separation of variables and coordinate axis transformation is presented which is also applicable for any value surface wave amplitude and frequency without any singularity in the solution in addition to expressing the frequency spectrum of acoustic signal.

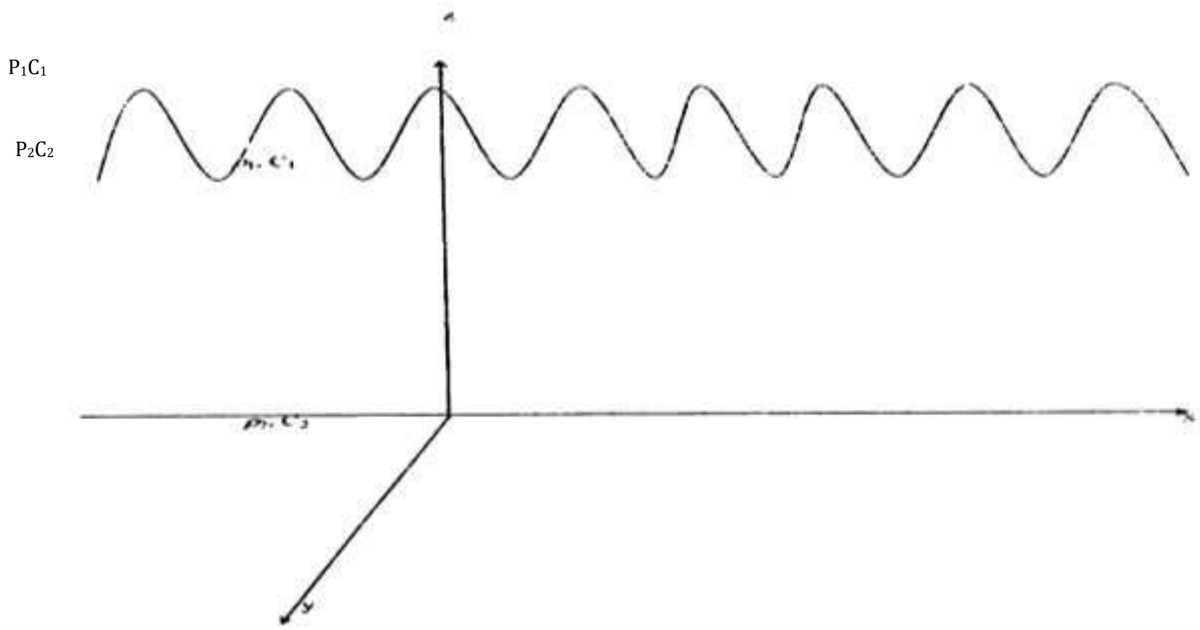

Figure – 1

2. THEORY: Consider an oceanic wave guide with a wavy surface as shown in the Figure-1. The surface wave is a single frequency wave propagating in x direction which can be expressed mathematically as

$Z = h(x, t) = h_0 + a \cos(\alpha x - \Omega t)$
(1)

Where h_0 is average channel depth, α is wave number, Ω is circular frequency and a is amplitude of the surface wave. Let ρi, Ci (i = 1,2) denote respectively the density and velocity of sound in the two media. Medium 2 is assumed to be a semi-infinite one. The interface between medium 1 and medium 2 is assumed to be flat.

Let us assume that a plane sound wave propagates in the wave guide in the direction x. The acoustic pressure P (x, y, z, t) can be determined by solving the wave equation.

$$\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} + \frac{\partial^2 P}{\partial z^2} - \frac{1}{C^2}\frac{\partial^2 P}{\partial t^2} = 0$$
(2)

with boundary conditions

P (x, y, z, t) = 0 at z = h (x, t) (3a)

P(x , y, + 0, t) = P (x, y, -0, t) (3b)

$$V_z\big|_{z=+0} = V_z\big|_{z=-0} = \frac{1}{\rho_1}\frac{\partial P}{\partial z}\bigg|_{z=+0} = \frac{1}{\rho_2}\frac{\partial P}{\partial z}\bigg|_{z=-0}$$
(3c)

where Vz is z component of particle velocity

P → 0 a z → - ∞
(3d)

Where +0 and -0 indicates that the interface z = 0 is approached from the sides z > 0 and z < 0 respectively. Since there is no variation of pressure in y direction, equation (2) can be written as

$$\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial z^2} - \frac{1}{C^2}\frac{\partial^2 P}{\partial t^2} = 0$$
(4)

The equation (4) can be written in the frequency domain as, if there is no change of acoustic frequency by the surface wave

$$\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial z^2} + k^2 P = 0$$
(5)

Where

$$k = \frac{\omega}{c_1} = k_1 \;\; for\; 0 < z < h$$
(6a)

$$= \frac{\omega}{c_2} = k_2 \;\; for \;\; z < 0$$
(6b)

Let X = h(x,t), where X is a a linear scale in the x dimension and so, $0<h<h_0$. So, the equation (5) becomes if the scale x is changed to X

$$\frac{\partial^2 P}{\partial X^2} + \frac{\partial^2 P}{\partial z^2} + k^2 P = 0$$

Therefore, by separation of variables methods,

$P(X, z, t) = \Psi_n(z).P_x(X)$ (7a)

$P_x(X) = \mathrm{Sin}(\zeta_n X)$ (7b)

The function $\Psi_n(z)$ must satisfy the equation

$$\frac{\partial^2 \Psi_n}{\partial z^2} + (K^2 - \xi_n^2)\,\Psi_n = 0$$
(8)

with the boundary conditions

$\psi_n = 0$ at $z = h(x, t) = X = h_0$ (9a)

$\psi_n(+0) = \psi_n(-0)$ (9b)

$$\frac{1}{\rho_1}\frac{\partial \psi_n}{\partial z}(+0) = \frac{1}{\rho_2}\frac{\partial \psi_n}{\partial z}(-0)$$ (9c)

$\Psi_n \rightarrow 0$ as $z \rightarrow -\infty$
(9d)

The solution to the equation (8) is

$\Psi_n = N_n \sin \chi_n (z/h_0 - 1)$ for $0 < z < h_0$
$= Cn\, e^{Dnz}$ for $z < 0$
(10)

Where N_n, C_n, χ_n & D_n, are constants, Ψ_n must satisfy the orthonormality condition.

$$\int_{-\infty}^{h} \rho(z)^{-1} \psi_n \psi_m \, dz = \delta_{mn}$$, $h = h_0$, Where $\rho(z) = \rho_1$ for $0 < z < h_0 = \rho_2$ for $0 < z$ and δ_{mn} is kronecker delta. (11)

By substituting the equating (10) into equation (1) and (8) we get

$$\int_0^h N_n^2 \rho_1^{-1} \sin^2 \chi_n\left(\frac{z}{h} - l\right) dz + \int_{-\infty}^0 \rho_2^{-1} C_n^2 e^{2Dnz} dz = 1$$, $h = h_0$ (12)

$$\xi_n^2 = K_l^2 - \left[\frac{\chi_n}{h}\right]^2$$, $h = h_0$
(13)

$$D_n = (\xi_n^2 - K_2^2)^{1/2}$$
(14)

Substitution of equation (10) into equations (9b) & (9c) gives,

$-N_n \sin\chi_n = C_n$ (15)

$$\frac{1}{\rho_1} N_n \left(\frac{\chi_n}{h}\right) \mathrm{Cos}\chi_n = C_n \frac{D_2}{\rho_2}$$, $h = h_0$ (16)

Equations (13) to (16) can be combined to get

Cot $\chi_n = -(q\,\chi_n)^{-1}\,(h^2(K_1^2 - K_2^2) - \chi_n^2)^{1/2}$ h=h$_0$
(17)

where q = ρ_2 / ρ_1 From equation (12) we get

$$N_n = \sqrt{\frac{2}{h\rho_1^{-1}(1-\frac{\sin 2\chi_n}{2\chi_n}) + \frac{\rho_1^{-1}\sin^2\chi_n}{D_n}}} \quad , h=h_0 \qquad (18)$$

χ_n can be found by solving the equation (17).

Thus the complete analytical solution of the problem is obtained. For $C_n = 0$, $\Psi_n = 0$ at z = 0. Therefore, sin $\chi_n = n\pi$, where n is a positive integer. The solution is P (X, z, t) = sin χ_n (z/h$_0$ – 1). Sin(ζ_n.X - ωt). (19)

3. EXACT SOLUTION TO WAVEGUIDE OF ANY DIMENSION: In the previous section, the direction of propagation of the acoustic wave and the surface wave were assumed to be in the same direction. If the direction of propagation of the acoustic wave is assumed to be different from that of the surface wave, the solution is
P (X, z, t) ≈ sin χ_n (z/h$_0$ – 1). Sin(ζ_n.X +β_ny- ωt), where β_n is the wave number of acoustic wave in the y direction. (20)

If a cylindrical acoustic wave is assumed to be propagating in the wave guide, the solution is
P(R, z, t) ≈ sin χ_n (z/h$_0$ – 1). Re.($H_0^1(\zeta_n R).e^{-jn\omega t}$) where H_0^1 is the Hankel function of 0th order and 1st kind and R = $(X^2 + y^2)^{1/2}$ (21)

The simple method presented here is very well valid for any type of surface wave whether it is a deterministic or random or periodic or non periodic or surface waves propagating in both x and y directions or modulated, etc.

The perturbation method used by Anand, et al and Sundaravadivel creates singularity. Therefore, accurate determination of field close to the singularity becomes very complicated. The method presented in this section does not generate any such singularity.

4. APPLICATION OF THE METHOD:
A.H.Nayfeh and O.H.Kandil [13] studied wave propagation in a circular cylindrical waveguide with sinusoidal boundary walls by applying perturbation techniques. The method presented in this article could be extended to study the wave propagation in such cylindrical waveguides with variable boundary. The boundary need not be periodic to apply the method. Similarly the method is applicable to study the wave propagation in the waveguides of A.M.Nusayr and M.A. Hawwa [14 15]. K.Srivatsava and Fabrizio Frezza, et al [16 17] had studied the wave propagation in 1-dimensional and two dimensional periodic waveguides. Both the variable boundary and the medium generate periodic waves in the propagating waves of waveguide without any boundary or medium variations. Therefore, variations of the medium could be equivalently transformed to variations in the boundary of the waveguide. After such transformation, the method developed in this article could be extended to find the field in the waveguide. More general waveguide models [18] needs numerical solution to the analytical expressions derived in this article.

5. CONCLUSION:
A much simpler analytical solution to the problem of sound wave propagation in an ocean with a wavy surface is derived which is applicable to surface wave of any amplitude and frequency without any singularity in the solution. The method could be extended to analyze the ionospheric radio wave propagation and also to analyze the light wave propagation in optical waveguides with surface irregularities.

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BIOGRAPHY

Dr. John Daniel is currently pursuing a D.Sc. degree of Mumbai University. He received a D.Litt.(Hon.) degree in 2010. He got his Ph.D. in 2013. He received his M.Sc.(Engg.) degree from the I.I.Sc., Bangalore in 1990. His first degree B.E. in E.C.E. was received in 1985 with distinction. So far, he has published over 85 research papers in the above mentioned fields and also two books. He worked as a researcher in the internationally reputed organizations like ET & T, New Delhi, T.I.F.R. Mumbai, I.I.Sc. Bangalore, University of Waterloo, Canada and R.R.I. Bangalore and also in various private engineering colleges and rose from a Lecturer to a Director.

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