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Operations of Complex Numbers on The Development of Students Mathematical Thoughts in Liberia

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Abstract: *This study aimed to improve the implementation of the Mathematics Curriculum in Senior Secondary Schools and develop students' Mathematical thoughts on the operations of Complex Numbers, especially in the Gardnersville belt, in Monrovia, Liberia. The research has three goals: 1. They were investigating students' mathematical knowledge of complex numbers, 2. Examining the extent to which students have improved their understanding of complex number representation, and 3. Improving students' understanding of complex number operations in Gardnerville. Because the study incorporates measurement, categorization, and analysis, the research design used in this study is a descriptive survey. The study included 120 mathematics teachers from 15 secondary schools in Gardnersville, Liberia. A qualitative research design was adopted in this study. This study gathered data via questionnaires. This design examined a large population, with only a small percentage of the population being used to provide the required data. The researcher used 15 secondary school sampling frames to sample the required number of schools for the study randomly. The study has a sample size of 120 participants. In this study, random sampling was used as a sampling technique. When a population participant has an equal probability of being chosen, random sampling is used. This is because each member of a population is assumed to possess all of the population's features. This study considered the teachers' gender, level of education, age, and work experience. However, the study's results regarding the study's objectives were critical. As a result, the findings are summarized as follows: In terms of background information, the results demonstrate that males represent the vast majority of respondents (95%) in the survey, showing that males dominated. The majority of the population was between the ages of 20 and 30 (47.9 %). A more significant percentage of respondents had Bachelor's degrees, indicating that they were more educated (42.5 %). Respondents with 7 years or more of experience represented the majority of the respondents (50.8 %).*

Key Words*: Complex Numbers, Operations, Development, and Mathematical Thoughts.*

1. INTRODUCTION:

The primary goal of mathematics education is to foster mathematical thinking and impart reasoning techniques that will aid students in solving problems both in secondary schools and at higher levels of learning and knowledge. (Jones, 2010). Solving various problems is critical for developing sound reasoning habits (NCTM, 2009); we argue that reasoning development can be accelerated even further by utilizing multiple methods for solving and locating a single problem. Studies in plane geometry can aid in the development of additional mathematical proofs and reasoning channels. (Chazan, 1993) and should be addressed in teacher education curricula. One objective of many countries' mathematics curricula, such as Liberia, Ghana, Nigeria, Guinea, Sierra Leone, Ivory Coast, America, Canada, and Australia, is to develop students' ability to think, prove, and reason. (Schoenfeld, 2016).

For example, in the United States, the following recommendations are made in Principles and standards for school mathematics (NCTM, 2000): Prekindergarten through grade 12 instructional programs should enable all students to:

i. recognize reasoning and proofs as fundamental aspects of mathematics;
ii. formulate and investigate mathematical conjectures;
iii. develop and evaluate mathematical arguments and proofs, and
iv. select and apply various types of reasoning and proof methods

To think mathematically, an individual must develop the ability to apply what has been learned as information and perform analysis and synthesis using previously taught basic operations, properties, rules, and theorems. "It required a renewed commitment to seeking application and solutions, rather than simply memorizing procedures; to exploring and justifying patterns, rather than merely memorizing formulas; and to formulating conjectures, rather than simply performing exercises. "(Schoenfeld, 2016). To foster such thinking and provide opportunities for students to develop mathematical thoughts in mathematics, teachers must possess a breadth and depth of pedagogical content knowledge that enables them to incorporate various problem-solving methods and strategies into their instruction. (Steele & Rogers, 2012)

The approach to mathematics education is critical in Liberia because most mathematics resources available to teachers and students are influenced by western culture. Lessons utilizing such resources are frequently deemed "impossible" or "impossible" by learners. They are exposed to a variety of visual representations throughout the teaching and learning process. At the most informal level, models may take the form of diagrams, photographs, and tables to organize their data. They are introduced to abstract mathematical symbols and representations on a formal level.

This topic was chosen to highlight some of the difficulties students face in secondary school when it comes to grasping the operations of Complex Numbers, their geometric representation, and their applications. This topic, which is included in Liberia's senior secondary mathematics curriculum, provides students with opportunities to deepen their understanding of Complex Numbers' operations and develop their mathematical thinking and problem-solving skills through the application of Complex Numbers. The purpose of teaching mathematics in Senior Secondary Schools is to prepare students with mathematical knowledge to apply their expertise in fields such as engineering and mathematics education. These individuals are preparing to meet society's living standards and the challenges it faces, its values, and hopes for the future. As a mathematics educator, I frequently hear students in Liberia ask, "Why do I need to know this?" Alternatively, "When will I ever use this?" Students want to understand how the mathematics they are studying relates to or affects their lives. These are critical questions to address, yet many mathematics teachers in Liberia's various secondary schools instruct students on solving problems rather than motivating them to think for themselves. In Liberia, for example, traditional mathematics instruction consists of a teacher working through a series of issues while students practice solving an assigned set of problems. This has created a difficult situation for students as they attempt to develop their mathematical thoughts and concepts. Regrettably, Liberia's approach to mathematics education has left many students capable of the mathematics behind.

Additionally, students have encountered numerous difficulties when it comes to complex numbers. i. Accepting the existence of a number i that satisfies i^2= -1. ii. Overcoming anxieties generated by the unfortunate use of "imaginary" and "complex" about specific numbers. iii. Confusion of algebraic, geometric, and trigonometric concepts; and (iv) ignorance of origin, history, and utility of complex numbers.

1.1. Background to the Study:

The approach to mathematics education is critical in Liberia because most mathematics resources available to teachers and students are influenced by western culture. Lessons utilizing such resources are frequently deemed "impossible" or "impossible" by learners. They are exposed to a variety of visual representations throughout the teaching and learning process. At the most informal level, models may take the form of diagrams, photographs, and tables to organize their data. They are introduced to abstract mathematical symbols and representations on a formal level. This topic was chosen to highlight some of the difficulties students face in secondary school when it comes to grasping the operations of Complex Numbers, their geometric representation, and their applications. This topic, which is included in Liberia's senior secondary mathematics curriculum, provides students with opportunities to deepen their understanding of Complex Numbers' operations and develop their mathematical thinking and problem-solving skills through the application of Complex Numbers.

1.2. Statement of the problem:

The purpose of teaching mathematics in Senior Secondary Schools is to prepare students with mathematical knowledge to apply their expertise in fields such as engineering and mathematics education. These individuals are preparing to meet society's living standards and the challenges it faces, its values, and hopes for the future. As a mathematics educator, I frequently hear students in Liberia ask, "Why do I need to know this?" Alternatively, "When will I ever use this?" Students want to understand how the mathematics they are studying relates to or affects their lives. These are critical questions to address, yet many mathematics teachers in Liberia's various secondary schools instruct students on solving problems rather than motivating them to think for themselves. In Liberia, for example, traditional mathematics instruction consists of a teacher working through a series of issues while students practice solving an assigned set of problems. This has created a difficult situation for students as they attempt to develop their mathematical thoughts and concepts. Regrettably, Liberia's approach to mathematics education has left many students capable of the mathematics behind.

Additionally, students should be utilizing mathematics applications to demonstrate what they have learned about a particular mathematical concept in a practical setting. In Liberia, students are unable to justify and explain their mathematical reasoning adequately. Mathematics applications assist students in absorbing, accumulating, and reproducing knowledge of mathematical concepts.

Students have encountered numerous difficulties when it comes to complex numbers. i. Accepting the existence of a number i that satisfies i^2= -1. ii. Overcoming anxieties generated by the unfortunate use of "imaginary" and "complex" about specific numbers. iii. Confusion of algebraic, geometric, and trigonometric concepts; and (iv) ignorance of origin, history, and utility of complex numbers.

1.3. Purpose of the study

The primary objective of this study is to improve the implementation of the Mathematics Curriculum in Senior Secondary Schools in the area of Complex Numbers and to provide empirical evidence regarding the achievement of the study's goals and objectives in Monrovia, Liberia, more precisely in the Gardnersville belt.

1.4. Research Objective:

The goal of this research is to;

- Investigate students' mathematical knowledge about complex numbers
- Examine the extent to which students have a better understanding of the representation of Complex numbers.
- Improve students' understanding of the operations of complex numbers.

1.5. Research Questions:

This research aims to address the following questions:

- Why were complex numbers introduced to develop students' mathematical thoughts?
- How well do students understand how complex numbers are represented?
- What ways can be used to improve students' understanding of the operations of complex numbers?

1.6. Significance of the Study:

The purpose of this study is to

- Provide teachers with effective teaching techniques and exercises for complex numbers in secondary school Algebra, Geometry, and Trigonometry.
- Introduce complex numbers to secondary school students to help them develop their mathematical thinking about them.
- Assist students in overcoming obstacles to accepting and comprehending complex numbers.
- Assist teachers in identifying strategies for enhancing students' understanding of complex numbers.
- Foster students' mathematical reasoning regarding the use, operations, and representation of complex numbers.

2. LITERATURE REVIEW:

Mathematical concepts have evolved historically, like how students develop cognitively. To begin, we examine humanity's capacity for comprehending complex numbers. Humanity's understanding process can be divided into three phases: introducing complex numbers, their geometric representation, and their application. As strange as it may sound, historical reality was far too dissimilar, strange, and illogical. Complex numbers developed and gained acceptance in lockstep with the development and approval of negative numbers. The successful introduction of complex numbers into mathematics also paved the way for creating a new class of numbers capable of resolving and explaining various problems, including hypercomplex numbers, sedition, hyper-real numbers, and surreal numbers. The complex number was discovered while attempting to solve specific exponent-based equations—these problems developed into genuine problems for mathematicians.

Although we did not find any empirical studies, we found several sources that addressed the historical development of complex numbers and hypothesized some of the cognitive milestones necessary for understanding complex numbers. Following that, we summarize several of the insights we gained from these readings. Each expansion of our concept of number can be explained similarly, with division motivating the need for rational numbers, taking square roots motivating the need for irrationals, and, later, the need for complex numbers motivating the taking of roots of negative numbers. (Fauconnier & Turner, 2008) the assertion that complex numbers could not have existed until they occurred between two mental spaces: real numbers, which support arithmetic operations, and vectors in the Cartesian plane, which support magnitude and direction. This procedure establishes the concept of a complex number as both a number and a vector. (Lakoff & Núñez, 2000) they use related conceptual blending ideas and apply the metaphors for a series of number and number operations to form their description of the conceptual development of complex numbers. The metaphor for the development of negative numbers was fascinating to us.

Researchers have explored students' (Conner et al., 2007); (Danenhower, 2000); (Nemirovsky et al., 2012); (Panaoura et al., 2006) as well as experts' (Soto-Johnson et al., 2011) Complex numbers and complex-valued functions have geometric representations and interpretations. By and large, findings indicate that experts regarded Complex numbers, operations, and complex-valued functions as objects effortlessly. This was not true, however, for students. (Conner et al., 2007) claim that Secondary mathematics teachers believe that multiplying an actual number by a negative number is a reflection rather than a 180-degree rotation. This belief may have developed due to their focus on the real number line rather than the entire complex plane, resulting in their inability to demonstrate how multiplication by the complex number x & yi results in the rotation and dilation of the other factor. Additionally, the pre-service teachers defined complex numbers as pairs of real numbers as opposed to a single number. This belief may have motivated their attempt to provide a geometric interpretation of complex number addition by employing vectors or by decomposing the numbers into real and imaginary segments.

Unfortunately, this belief in complex numbers does not relate to the geometric interpretation of complex number multiplication. (Nemirovsky et al., 2012), provided procedures that established that rotating 90 degrees counter clockwise is equivalent to multiplying by i. This was an experiment in teaching with pre-service secondary teachers. The classroom floor served as the complex plane, and participants engaged in physical explorations of multiplying behaviour. This type of perception-motor activity created an environment in which participants discovered and conceptualized complex addition and multiplication.

However, to utilize such instructional strategies, teachers need to know various representations and connections among them. (Ball et al., 2001) discussed how an elementary teacher diagnosed students' misunderstanding of decimal number multiplication due to inappropriate use of an area model representation and developed a lesson to address the misconception. Ball accentuated that the development and delivery of the study illustrate the teacher's ability to connect students' representations to mathematically correct ones and acknowledged that it required considerable mathematical depth and flexibility of the content. Similarly, (Izsák & Sherin, 2003) documented how teachers' understanding of multiple representations allowed them to assess students' misunderstandings and use them as a pedagogical tool. (Lesh et al., 1987), (Ball et al., 2001) and (Izsák & Sherin, 2003) stated why teachers need to know different representations of concepts in their teaching and individual problem-solving activities. Good problem solvers are aware of which representation is appropriate for a given task. Therefore, it is imperative teachers be knowledgeable about integrating multiple representations while communicating concepts to students. This study contributes to the literature related to the Operations of Complex Numbers on developing students' mathematical thoughts while working with different forms of Complex Numbers.

2.1. History of Complex Numbers:

The historical development in mathematics occurred due to ontological turns that brought structural symbols into a discourse concerned with the operation. This happened when signs such as "i" were proposed. Complex Numbers came into existence earlier than that. Mathematicians depended solely on symbol systems to help in the process of their reasoning. They try to develop symbol systems that capture essential aspects of their reason and means of operation. The knowledge about the mathematical development of the number system arose by the emergence of new numeric understanding, such as the understanding that the number system could be created even if no one understood what they were, as in the case of Complex Numbers (Henle, 1986). The Complex Numbers were created by symmetrically operating on the symbols without regard for their meaning. The Complex Number (-1) was discovered during mathematicians' efforts to solve problems. In particular, a famous Italian mathematician Cardano (1501-1576), tried to extend Tartaglia's method for solving the equation of the form $x^3 + px = q$. During his manipulation he extracted the next square root $W = \sqrt{(\frac{1}{2}q)^2 \text{-} (\frac{1}{3}p)^3}$.

The difficulty is evident in the case that we have in mind only positive numbers. The problem arises if the difference under the root becomes negative. He called it 'casus irreducibles.' It is thought that the Italian mathematicians Cardano and Bombelli in the 16th century were among the first to utilize Complex Numbers by calculating with a quantity whose square was -1. Since then, other scholars have modified the original definition of the product of Complex Numbers. The geometer Glifford (1845-1879) developed the 'double' Complex Numbers by requiring that i^2=1. In 1637, Descartes originated the term 'real and 'imaginary,' and in 1748, Euler introduced the letter' i' to represent the concept. Even though Gauss used a method of graphing Complex Numbers, by 1811, he had decided the study needed a formal foundation, i.e., a standard set of postulates from which the arithmetic properties of Complex Numbers could be deducted.

Since Gauss proves the Fundamental Theorem of Algebra, we know that all Complex Numbers are of the form' x+yi,' where x and y are real numbers, real numbers being all those numbers that are positive, negative, or zero. Therefore, we can use the XY -plane that way. That brought about the ways to represent Complex Numbers; the first way Complex Numbers were represented was algebraical, as in the expression x+yi. Real numbers are considered exceptional cases of Complex Numbers; they are just x+yi when y is 0; they are numbers on the real axis. Euler visualized Complex Numbers by a geometric representation in polar coordinates r and @, and he wrote a formula, also known by Bernoulli: x+yi = $r\ (cosf\ + sin@)$. Developing the geometrical interpretation of Complex Numbers in a plane took a very long time.

Complex Numbers as a geometric entity and, more specifically, the representation of Complex Numbers in a plane was introduced in 1673 by John Wallis. He came very close in representing the point (a, b) with the complex quantity a+bi, but the idea of the perpendicular axis to the real axis for imaginary numbers with the standard unit $\sqrt{-1}$ was failed to have been introduced by him. Caspar Wessel was the one who introduced this idea in 1797-1798. Wessel's work made some significant points that brought about the definition of operations with lines. From 1799-1831, Gauss proposed the representation of Complex Numbers as points in the plane, rather than as directed line segments, as Wessel and Argand did. He considered the number a+bi as the point (a, b).

The geometric representation of Complex Numbers became well known by 1830 through the work of Wessel, Argand, and Gauss. They also recognized that vectors give a physical entity to Complex Numbers, which were only symbolical representations until then. In the 18^{th}-century, Complex Numbers gained wider use, as it was noticed that formal manipulation of Complex expressions could be used to simplify calculations involving trigonometric functions. For example, Abraham de Moivre demonstrated in 1730 that the complicated relationships between trigonometric functions of an integer multiple of an angle and powers of trigonometric functions of that angle might be re-expressed using the well-known de Moivre's formula.

2.2. Study of the Impact of Complex Numbers on Student Mathematical Thoughts:

It seems reasonable to provide future mathematics teachers with appropriate opportunities to engage actively in mathematical topics and tasks during their training. This should include routine tasks that become more or less difficult as the lecture progresses and open and self-differentiating tasks. In other words, their teacher college or university education should prepare them for and reflect, to the extent possible, what they can expect in their future careers. Empirical evidence supports the view that teachers who have learned and engaged in mathematical activities are better equipped to promote and facilitate their students' mathematical thinking. Numerous issues about teachers' subject-matter expertise are heavily linked to pedagogical content understanding and effective classroom practice. (Shulman, 1986). The impact of students' learning and operations with complex numbers has shifted students' conceptions of a complex number. This chapter discusses data that cast doubt on some assumptions about how students' conceptions of the complex plane develop. Our initial motivation to study complex numbers stemmed directly from our experience instructing prospective secondary school mathematics teachers. It is critical in the teaching of complex number fundamental operations. For example, consider the multiplication of two complex numbers. $r1(cos\ \theta1\ +\ i\ sin\ \theta1)$, and $r2(cos\ \theta2\ +\ i\ sin\ \theta2)$ can be taught as locating the point that is r1r2 distance from the origin and subtends an angle of θ1 + θ2 for the positive x-axis, measured counter clockwise. Thus, multiplication results in a transformation involving rotation, expansion or contraction, and rotation. Students may be taught the topic using both algebraic and geometric methods. Teachers can use the vector method to demonstrate complex addition and subtraction because students see complex numbers represented visually in the Argand plane.

However, it does not help teach multiplication and division. It is recommended that exercises be based on historical problems to learn with a heightened sense of motivation. When mathematicians have difficulties understanding complex numbers, they feel they are not alone in viewing the square roots of negative numbers with suspicion and skepticism. This feeling of inclusion goes a long way in developing their interest in Mathematics. Exciting problems can be given to students that make them think and appreciate complex numbers. For instance, a situation such as "Solve x^3 -7x+6= 0 or “Solve x^4 = -81" enables them to reason about and discuss mathematics. Students should be taught that by introducing complex numbers, any polynomial equation can be solved. They will need to develop activity and creativity through various interactions and learning experiences to impact learning complex numbers significantly. Student engagement is a critical component of the learning process's success. Activeness is a state of physical and mental activity in which one acts and thinks in unison. Learning to navigate through a variety of activities successfully requires both physical and psychological training.

2.3. Teachers Attitude Towards Incorporating Learning Activities in Mathematics Lessons:

Teachers must incorporate instructional activities into their mathematics lessons. Whether teachers choose mathematics as a career or not, whether they are comfortable with their mathematics workload, or whether their salaries are low, they must exhibit specific characteristics of a mathematics teacher. Teachers should always maintain a favorable attitude toward educational activities. Teachers should always have a positive attitude and expression toward learning activities when presenting mathematics lessons. (Nafungo, 2004); Take note that an overburdened mathematics curriculum affects teachers' methods. Mathematics is taught theoretically to cover the syllabus, and students are not given time to discover things for themselves. This makes mathematics difficult and boring to learn. When students are unable to enjoy or admire mathematics lessons, a negative attitude toward the subject develops.

This situation will eventually result in a loss of interest in the subject and a low level of achievement. The following are a few strategies teachers can use to help students understand mathematics in the classroom: 1. Develop an engaging class opener 2. Introduce students to the topics by utilizing a variety of different concepts and representations. 3. Demonstrate to students' numerous additional steps necessary to solve a mathematical problem. 4. Demonstrate to students the relevance of the problem to their lives. 5. Encourage students to use their brains and communicate their reasoning, and 6. The class should conclude with a conclusion. The teacher's attitude toward students learning mathematics in the classroom significantly impacts students' goal-setting abilities, problem-solving abilities, beliefs about mathematics, inner and external motivations for mathematics learning, and other academic performances. When you envision yourself as a teacher, your attitude is critical. Your attitude as a teacher has various effects on your students and can also shape their educational experiences and careers.

As a mathematics teacher, you may occasionally encounter stress and distraction from your students, as many are uninterested in the subject. Always look for a positive way to alleviate your stress and distractions as a professional and experienced teacher. A few teaching strategies will help you improve your class as a proficient mathematics teacher: 1. As an experienced educator, you should always differentiate your instruction based on the needs of your students. Differentiating instruction strategies enable a skilled teacher to engage each student by catering to their unique learning style. 2. Students should work cooperatively in the classroom. 3. Teachers should incorporate technology into the classroom in the twenty-first century. 4. Teachers' instruction should be inquiry-based. A positive attitude positively affects a student's motivation, attitude toward school and schoolwork, self-esteem, and personality development. (Gundogdu & Silman, 2007).

3. METHOD:

This aspect highlights the research design, setting, population, sample & sampling techniques, data sources, instruments used to collect the data, validity, and reliability, methods used to analyze the data collected, and ethical considerations.

3.1. Research Design:

A research design is an arrangement of conditions for collecting and analyzing data in a manner that aims to combine relevance to the research purpose (Kothari, 2004). The research design constitutes the blue point for the collection, measurement, and analysis of the data. The research design also served as the conceptual structure within which the research is conducted. The research design used in this research is a descriptive survey because the study involves measurement, classification, analysis, comparison, and interpretation. The descriptive analysis method is used to analyze data to describe or illustrate data that has been collected. This study aims to explore the operations of complex numbers on the development of students' mathematical thoughts in Gardnersville, Liberia. The study focused on 15 selected secondary schools with 120 mathematics teachers across Gardnersville, Monrovia, Liberia. This study is involved with a qualitative research design.

During this study, information was collected through the means of questionnaires. This design allowed for a large population study while just a sample of participants provided the necessary data. Thus, the researcher will be able to assess the teacher's involvement success in developing students' mathematical thinking about complex numbers, the effect of students' readiness to accept and understand complex numbers, the teacher's teaching techniques and exercises for assisting students in developing a better understanding of the development, representation and, operations, and application of complex numbers, and the teacher's exploration of some methods.

3.2. Sample and Sampling Technique:

(Slavin, 1984), observes that due to time, funds, and energy limitations, a study can be carried out from a carefully selected sample to represent the entire population. Sampling is a process of representatives of the target population. Sampling begins by dividing the population into relevant sub-groups and then random sampling from each sub-group. The researcher used 15 secondary schools sampling frames to randomly sample the required number of schools to participate in the study. The participants comprised 120 people. Random sampling was used in this study. Random sampling is the technique employed when a population participant has an equal chance to be selected. This is because each participant of the population is assumed to have all the characteristics of the population. Random sampling was used to determine teachers from each of the 15 secondary schools across Gardnersville, whether male or female.

3.3. Research Instrument:

The research questionnaire was the instrument used for collecting data. The online questionnaire ensures the accuracy of the information that the respondents completed at their convenience. (Creswell, 2002), a questionnaire is a document that respondents fill out and return to the researcher as part of a survey design. Additionally, the research questionnaire guarantees confidentiality. The questionnaire was generated via the Microsoft survey platform and contained a total of 22 questions. These questions were subdivided into two main sections. Section 'A' was about the respondents' background information, which included a few open-ended questions and was analyzed using descriptive statistics. Section 'B' was about teachers understanding complex numbers using the format of the closed-ended questions on a five-point Likert scale. The instrument used for this research analyzed the participation of 120 teachers of the 15 secondary schools in Gardnersville, Monrovia, Liberia, through online questionnaires. In terms of qualitative data gathering, this research uses sources from teachers in secondary schools in Gardnersville, Monrovia, Liberia. Qualitative data constituted the majority of the sources referenced in this research work. The researcher collected aimed at investigating the problem, followed by an analysis of data, which will lead to the interpretation of results and drawing of conclusion and policy recommendation. The first section of the questionnaire asked teachers about their sex(gender), level of education, age range, and work experience. The second section investigated the teacher's personal experiences, and the last section explored the teacher's understanding of Complex Numbers.

3.4. Validity and Reliability of the Instrument:

(Orodho, 2005)**,** defines validity as the degree to which results obtained from the data analysis represent the phenomenon under study. At the same time, reliability is the instrument's consistency in measuring what it is intended to measure. The validity, therefore, checks if the research instrument is doing what it was designed to do. To ensure validity, experts in the field of mathematics education had thoroughly reviewed the instrument. The instrument used in this student was an online questionnaire, and SPSS was used to compute the data collected from respondents.

3.5. **Data Collection Procedure:**

The researcher seeks the kind permission of respondents and explains the purpose of the study to each respondent via text messages and internet calls. Subsequently, the questionnaire link was sent via preferred internet platforms to respondents in Gardnersville, Monrovia, Liberia, to complete and submit the electronic questionnaires at their convenience. The researcher achieved a response rate of 100% due to the flexible nature of the submission and pre-contacting each respondent. All responses were received within three weeks.

3.6. **Data Analysis:**

Data collected from respondents were edited, coded, and analyzed statistically. Qualitative data collected using a questionnaire were analyzed using descriptive statics using the statistical (SPSS) and presented through percentage, mean, cumulative percent, and frequencies. The information was displaced by the use of bar charts, tables, and pie charts. The data analysis obtained from respondents was recorded in readiness for analysis. This was accomplished by tallying responses, determining response frequencies, percentages, and describing and interpreting the data following the study objectives using SPSS. The respondents were asked to rank the relevance of each aspect on a 5-point Likert scale ranging from 1 to 5, with 5 representing Strongly agree, 4 representing Agree, 3 representing Neutral, 2 representing Disagree, and 1 representing strongly disagree. The Relative Importance Index was computed using the following equation: $RII=\Sigma W/A*N$

Where:
W – scale for rating a factor (ranges from 5 to 1);
A – The highest weight on the scale;
N – Total number of respondents.
By implication, the Frequencies (N), Means (M), Scale for ranking a factor (W), Constant (A*N) were computed, and their numerical values and ranking gave the direction of the response to each research question and teachers' understanding about complex numbers.

3.7. Ethical Consideration:

To adhere to research ethics, respondents' permission was sought, and their confidentiality was guaranteed. Additionally, the study's primary objectives were explained to each respondent, ensuring that respondents participated voluntarily.

4. DISCUSSION:

4.1. Introduction:

The primary purpose of this study is to better implement the Mathematics Curriculum at the Senior Secondary Schools in the area of Complex Numbers and to provide empirical evidence on the attainment of its goals and

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objectives in Monrovia, Liberia, specifically in the Gardnersville belt. The results were presented based on the aim of the study and research questions, which aimed at determining the operations of complex numbers on the development of students' mathematical thoughts in secondary schools in Gardnersville, Monrovia, Liberia. This study sought information from teachers using questionnaires. A total of 22 questionnaires were presented to respondents, and 120 respondents responded to the questionnaire. Data analysis was done through descriptive statistics, and findings were presented in frequency tables and percentages. This chapter presents the summary of the analyzed data. The discussion of the outcomes was based on Statistical Packages for Social Sciences (SPSS) outputs.

4.2. Gender Distribution of Respondents:

This question sought to know the gender distribution of respondents. The Bar- Chart below shows that the vast majority of respondents are males representing 95%, while the remaining 4.2% are females; this indicates the dominance of males in the sector. One of the respondents mistakenly selected male and female at the same time while responding to the questionnaire and accumulated at 0.8%.

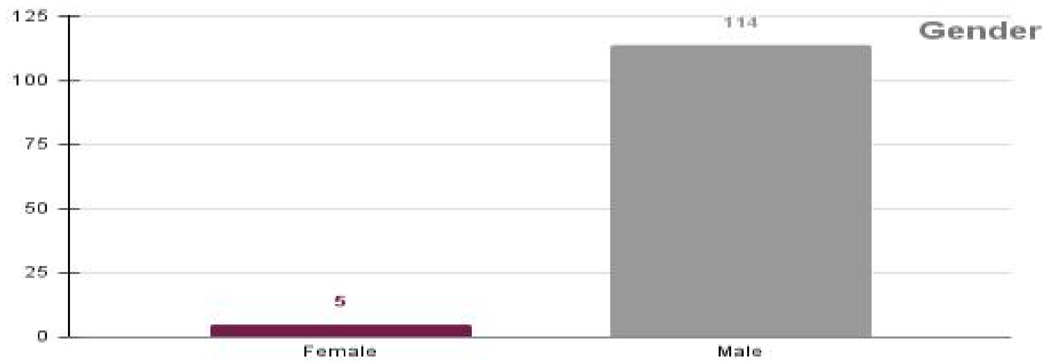

Figure 4.1: Gender Distribution of Respondents Represented in Bar Chart
Source; Field Survey, 2021

4.2.1. Age Distribution of Respondents

This question sought to identify the age distribution of respondents in secondary school in Gardnersville, Monrovia, Liberia. According to figure 4.2, 47.9% of teachers are between the ages of 20 and 30, 34.5% are between the ages of 31 and 40, and 17.6 % are in the range of 41 years and above. This indicates that the dominants of teachers are in the range of 20 – 30 years of age. This suggests a need for capacity replacement in the long term, as the 17.6% group is expected to retire in a few years. This research's respondents' ages determine the age categories in secondary schools in Gardnersville, Monrovia, Liberia. The idea here is that a person's age is likely to influence his knowledge of experience directly.

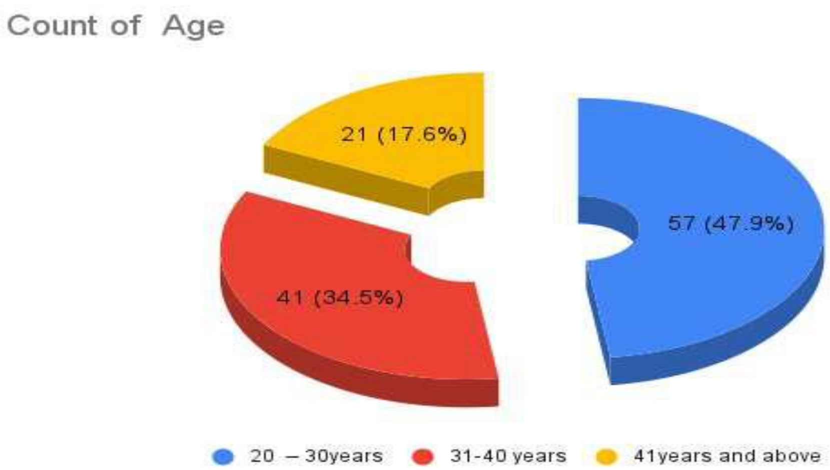

Figure 4.2: Age Distribution of Respondents Represented in Pie Chart
Source; Field Survey, 2021

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4.2.2. Academic Qualification Distribution of Respondents:

From table 4.1 below, 42.5% (51 out of 120) of the respondents holds BSc, 23.3% (28 out of 120) of the respondents holds Diploma, 17.5% (21 out of 120) of the respondents holds a certificate, 11.7% (14 out of 120) of the respondents holds MSc, 5% (6 out of 120) of the respondents holds WASSCE. The educational level of the respondents is significant since it influences the way to manage construction activities.

Table 4.1: Academic Qualification Distribution of Respondents

Qualification	Frequency	Percentage	Cumulative Percentage
BSc	51	42.5	100.0
Diploma	28	23.3	57.5
Certificate	21	17.5	34.2
MSc	14	11.7	16.7
WASSCE	6	5	5.0
Total	**120**	**100.0**	-

Source; Field Survey, 2021

4.3. Research Question One: Why were complex numbers introduced to develop students' mathematical thoughts?

The goal of the first study question was to Investigate students' mathematical knowledge about complex numbers.

Table 4.2 below shows five (5) identities of teachers' that can influence the development of students' mathematical thoughts through teachers facilitating their students to have a significant impact on learning complex numbers in senior secondary schools; teachers will have to develop activity and creativity through various interactions and learning experiences. Among these are 27 respondents who were ranked first (1st) with a mean value of 0.675. 42 respondents followed this ranked 2nd with a mean of 0.700, 13 respondents ranked 3rd of the mean value of 0.433, 7 respondents ranked 4th of the mean value of 0.292, and 31 respondents ranked 5th with a mean of 0.258 respectively.

Table 4.2: Mean Score Rank for Identification of teachers' understanding about impacting their student's knowledge

Likert Scale	Frequency	ΣW	A*N	Mean(ΣW/)	RII (ΣW/A*N)	Ranking
Strongly Agree (5)	7	35	600	0.292	0.058	4th
Agree (4)	13	52	600	0.433	0.087	3rd
Neutral (3)	27	81	600	0.675	0.135	1st
Disagree (2)	42	84	600	0.700	0.140	2nd
Strongly Disagree (1)	31	31	600	0.258	0.052	5th
Total(N)	**120**	-	-	-	-	-

Source; Field Survey, 2021

4.4. Research Question Two: How well do students understand how complex numbers are represented?

The goal of the second study question was to examine the extent to which students have a better understanding of the representation of Complex numbers.

Table 4.3 below shows five (5) identities of teachers' that can influence students' understanding of the representation of complex numbers in the area of Algebra, Geometry, and Trigonometry. Among these are 42 respondents who were ranked first (1st) with a mean value of 0.700. 27 respondents were ranked second with a mean of 0.675, 14 respondents were ranked third with a mean of 0.583, 9 respondents were ranked fourth with a mean of 0.300, and 28 respondents were ranked fifth with a mean of 0.233.

Table 4.3: Mean Score Rank for Identification of teachers' understanding about the representation of complex numbers in Algebra, Geometry, and Trigonometry.

Likert Scale	Frequency	ΣW	A*N	Mean(ΣW/N)	RII (ΣW/A*N)	Ranking
Strongly Agree						

(5)	14	70	600	0.583	0.117	3rd
Agree (4)	9	36	600	0.300	0.060	4th
Neutral (3)	27	81	600	0.675	0.135	2nd
Disagree (2)	42	84	600	0.700	0.140	1st
Strongly Disagree (1)	28	28	600	0.233	0.047	5th
Total(N)	**120**	-	-	-	-	-

Source; Field Survey, 2021

4.5. Research Question Three: What ways can be used to improve students' understanding of the operations of complex numbers?

The goal of the third study question was to Improve students' understanding of the operations of complex numbers.

Table 4.4 below shows five (5) identities of teachers' that can influence students' understanding of the Operations of Complex Numbers on the development of students' mathematical thoughts utilizing teachers introduces complex numbers to their students in secondary schools by motivating them with the origin, history, and usefulness of complex numbers and later move over to perform basic operations of complex numbers (addition, subtraction, multiplication, and division). Among these are 43 respondents who were ranked first (1^{st}) with a mean value of 0.717. 21 respondents followed this ranked 2^{nd} with a mean of 0.523, 12 respondents ranked 3^{rd} of the mean value of 0.400, 9 respondents ranked 4^{th} of the mean value of 0.375, and 35 respondents ranked 5^{th} with a mean of 0.292 respectively.

Table 4.4: Mean Score Rank for Identification of teachers' understanding about the Operations of Complex Numbers on Students' Mathematical thoughts by performing Basic Operations (Addition, Subtraction, Multiplication & Division).

Likert Scale	**Frequency**	**ΣW**	**A*N**	**Mean(ΣW/)**	**RII (ΣW/A*N)**	**Ranking**
Strongly Agree (5)	9	45	600	0.375	0.075	4th
Agree (4)	12	48	600	0.400	0.080	3rd
Neutral (3)	21	63	600	0.525	0.105	2nd
Disagree (2)	43	86	600	0.717	0.143	1st
Strongly Disagree (1)	35	35	600	0.292	0.058	5th
Total(N)	**120**	-	-	-	-	-

Source; Field Survey, 2021

5. FINDINGS :

5.1. Introduction:

This aspect summarizes significant findings, recommendations, and conclusions meant to explore complex numbers' operations on developing student mathematical thoughts in Monrovia, Liberia. The case of secondary school students in Gardnersville. The primary purpose of this study is to better implement the Mathematics Curriculum at the Senior Secondary Schools in the area of Complex Numbers and to provide empirical evidence on the attainment of its goals and objectives in Monrovia, Liberia, specifically in the Gardnersville belt. The following objectives guided this study;

- To Investigate students' mathematical knowledge about complex numbers
- To Examine the extent to which students have a better understanding of the representation of Complex numbers.
- To Improve students' understanding of the operations of complex numbers.

5.2. Summary of the Findings:

The gender of the teachers, their degree of education, age, and job experience were all factors considered in this study. The study's findings of the study's aims, on the other hand, were of paramount relevance. As a result, the following summarizes the findings: In terms of background information, the study shows that males make up the vast majority of responders (95%), showing male dominance in the study. Those between the ages of 20 and 30 make up most of the population (47.9 %). A more significant proportion of the respondents held Bachelor's degrees, indicating (42.5 %). Respondents with experiences of 7 years and above represented (50.8%). The survey determined that teachers who taught 1-2 subjects were more dominant and competent in mathematics instruction. In contrast, most teachers taught less than 30 periods and were not administrators, indicating a light workload. Additionally, most students took ten or more subjects and were seated between 35 and 45 in class. In terms of student performance, the study found that most students received a weekly test, one to five mathematics exercises per class, and demonstrated strong academic performance.

5.3. RECOMMENDATIONS:

The following recommendations were made based on the findings of this study:

- **a)** First and foremost, through the Ministry of Education, the Liberian government should oversee all private schools and implement ongoing measures to reduce the maximum number of lessons per week and teaching period so that teachers have adequate time to prepare and deliver their lesson content effectively and efficiently.
- **b)** Secondly, through the Ministry of Education, the Liberian government should continue to monitor the implementation of the mathematics curriculum and encourage the mounting of in-service training and education on pedagogy and attitude change in mathematics for practicing teachers in Gardnersville, Monrovia, Liberia.
- **c)** Thirdly, the school administration should examine teacher-student interaction, which is crucial in determining students' performance.
- **d)** Fourthly, to avoid learning and teaching disruptions, the government and other educational stakeholders should provide secondary schools in the County with sufficient capital, learning materials, and human resources, distributed to schools on time, based on each school's student population.
- **e)** Furthermore, under the supervision of the Ministry of Education, the Liberian government should ensure that secondary school teachers receive frequent refresher workshops in mathematics.
- **f)** Additionally, in coordination with administrators, school owners should make every effort to hire more qualified and competent teachers in the subject areas and pay them a good salary and time for teachers to be effective and efficient.
- **g)** Moreover, to motivate students, the Ministry of Education should strive to offer deserving students who consistently perform well in the sciences in partnership with school operators.

5.4. CONCLUSION :

This study investigated the impact of introducing learning activities into Mathematics classes in Gardnerville secondary schools. This was about teachers' experiences in developing students' concepts and skills through complex numbers operations. The study concludes the majority of Mathematics teachers in Gardnerville secondary schools were inefficient and ineffective in their teaching methods and practices. Teachers should educate students on solving critical problems and how relevant the problem is to their lives. Teachers primarily instructed students by guiding them through examples and activities in a textbook that served as the coursebook. In addition to the book, standard teaching tools such as charts, real objects, models, and math laboratories should be used during mathematics lessons. Furthermore, some teachers had a negative attitude toward adding learning activities during Mathematics lessons. Some students had a negative attitude toward Mathematics lessons due to not being interested in Mathematics and teacher-related issues. Additionally, from the findings and recommendations of the study, it was concluded that: Teachers should adopt a new teaching method, employ techniques for assessing student performance and effectively implement the Mathematics Curriculum in secondary schools. The findings also show that most teachers teaching mathematics in secondary schools in Gardnerville had Bachelor's degrees, more years of teaching experience, and fewer subjects with fewer periods, and the majority were non-administrators.

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