



BULK VISCOUS FRW COSMOLOGICAL MODELS WITH FUNCTIONAL RELATION OF HUBBLE PARAMETER

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Abstract: We have studied time-dependent nature of the cosmological term Λ of the Einstein Field Equation in FRW cosmological models. We found that the presence of bulk viscosity is to increase the value of matter density ρ and to decrease the value of cosmological term Λ . This should aid resolution of several difficult problems of astronomy such as the best value for the Hubble parameter at present and at recombination. A detailed study of physical and kinematical behavior of the model is also discussed.

Key Words: FRW Model, Deceleration Parameter, Bulk Viscosity, Hubble parameter.

1. INTRODUCTION :

The field equations are calculated and analyzed separately for different epochs, although some worker have given unified solutions. For instance, Carvalho[1] studied Friedmann–Robertson–Walker (FRW) model for perfect fluid in general relativity and presented a unified solution using variable adiabatic parameter ‘gamma’ of ‘gamma-law’ equation of state. In the present paper we discuss the effects of bulk viscosity on the early evolution of universe for flat FRW model. The matter filling the cosmological (isotropic and homogeneous) background is functional relation on Hubble parameter discussed by a bulk viscous fluid having ‘equation of state and whose viscosity coefficient ζ is function of time T of the form $\zeta = \frac{1}{\zeta_0 + T}$, where ζ_0 is a positive numerical constant. We study the evolution of the

universe as it goes from an inflationary phase to a radiation-dominated era. Our approach is based on Carvalho’s [1] that studied the FRW models using functional form of gamma, depends on scale factor and presented a unified solution for two early phases of universe. Barrow [2] and Santos *et al* [3] studied FRW models with bulk viscous term. But both of them have given the solutions using equation of state with constant gamma.

Ruban and Finkelstein [4], Barker [5], Banerjee and Santos [6, 7] and Santhi & Rao [8, 9] are some of the worker who have studied several aspects of Nordtvedt general scalar-tensor theory. Rao and Kumari [10] have discussed a cosmological model with negative constant deceleration parameter in this theory. Rao *et al.* [11] have obtained Kaluza-Klein radiating model in a general scalar-tensor theory of gravitation. Rao *et al.* [12] have considered Kantowski-Sachs string cosmological model with bulk viscosity in general scalar-tensor theory of gravitation. Rao and Neelima [13] have studied Bianchi type-VI₀ space time with strange quark matter attached to string cloud in general scalar-tensor theory. Recently, Rao *et al.* [14] have studied Kantowski-Sachs dark energy cosmological model in general scalar-tensor theory of gravitation.

In this paper, we have studied FRW cosmological model with constant deceleration parameter in the presence of bulk viscosity in the scale covariant theory of gravitation.

2. METRIC AND FIELD EQUATION :

We consider homogeneous and isotropic spatially flat Rabertson-Walker line element of the form

$$ds^2 = dt^2 + S^2(t)(dx^2 + dy^2 + dz^2) \quad \dots (1)$$



where $S(t)$ is the scale factor.

The energy momentum tensor for bulk viscous fluid is taken as

$$T_{ij} = (\rho + p) v_i v_j + \bar{p} g_{ij} \quad \dots (2)$$

where ρ is proper energy density and \bar{p} is the effective pressure given by

$$\bar{p} = p - \xi v^i_{;i} \quad \dots (3)$$

satisfying equation of state

$$p = (\omega - 1)\rho; \quad 1 \leq \omega \leq 2 \quad \dots (4)$$

In the above equation p is the isotropic pressure and v^i is the four velocity vector satisfying $v^i v_i = -1$.

The Einstein field equations (in gravitational units $8\pi G = C = 1$) and varying cosmological constant $\Lambda(t)$, in comoving system of coordinates lead to

$$\bar{p} - \Lambda = (2q - 1)H^2, \quad \dots (5)$$

$$\rho + \Lambda = 3H^2 \quad \dots (6)$$

In the above equation, H is the Hubble parameter and q is the deceleration parameter defined as

$$H = \frac{\dot{S}}{S}, \quad \dots (7)$$

$$q = -1 - \frac{\dot{H}}{H^2} = \frac{-S\dot{S}}{\dot{S}^2} \quad \dots (8)$$

where an overhead dot ($\dot{}$) represents ordinary derivative with respect to t . The vanishing divergence of Einstein tensor gives rise to

$$\dot{\rho} + 3(\rho + \bar{p})H + \dot{\Lambda} = 0 \quad \dots (9)$$

3. SOLUTION OF FIELD EQUATION :

The equation (4) – (6) are three equations in five unknown variables s , ρ , Λ and ξ . Therefore, two more relation connecting these variables is required to solve these equation of system.

First, we assume that the law to be examined for the variation of Hubble's parameter

$$H = as^{-m}, \quad \dots (10)$$

where $a > 0$ and $m \geq 0$ are constants yields

$$S = [ma(t + t_0)]^{\frac{1}{m}}, \quad \dots (11)$$

where t_0 is constant of integration

let $t + t_0 = T$ then

$$S = (maT)^{1/m} \quad \dots (12)$$

From equations (8) and (11), we get



$$q = m - 1 \quad \dots (13)$$

Secondly, we assume

$$\xi = \frac{1}{\xi_0 + T} \quad \dots (14)$$

From equations (1) and(12), we have

$$ds^2 = -dt^2 + (maT)^{\frac{2}{m}} (dx^2 + dy^2 + dz^2) \quad \dots (15)$$

Expansion scalar θ , matter density ρ , cosmological term Λ for the model (14) is given by

$$\theta = \frac{3}{mT} \quad \dots (16)$$

$$\rho = \frac{1}{(w-1)} \left[\frac{3}{mT(\xi_0 + T)} + \frac{(2m-3)}{m^2 T^2} \right] \quad \dots (17)$$

$$\Lambda = \frac{3}{m^2 T^2} - \frac{1}{(w-1)} \left[\frac{3}{mT(\xi_0 + T)} + \frac{2m-3}{m^2 T^2} \right] \quad \dots (18)$$

We observe that the model starts with big-bang from its singular state at $T = 0$ and expansion in the model decreases with time increases. At $T = 0$, ρ , Λ , θ all diverge but ξ and ρ / θ^2 are constant at $T = 0$. In the limit of large time (i.e. $T \rightarrow \infty$), ρ , Λ , θ and ξ become zero but ρ / θ^2 is constant.

In the presence of bulk viscosity the value of ρ increases and the cosmological parameter Λ decreases.

4. CONCLUSION :

In this paper, I investigate FRW cosmological models with bulk viscous fluid and variable cosmological term Λ . I have solved exact solution of Einstein's field equations assuming variation of Hubble's parameter $\Lambda = as^{-m}$, where $a > 0$ and $m \geq 0$ are constant. We observe that $q = m - 1 > 0$, the universe is decelerating through the evaluation. When $m = 1$, we get $H = 1/T$ and $q = 0$. Therefore, galaxies move with constant speed. The model has point type singularity at $T = 0$.

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