



## Constant ratio of bulk viscosity and matter density in FRW cosmological models

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**Abstract:** We have considered Constant Ratio of bulk viscosity and matter density in FRW Cosmological Models in general relativity. The deceleration parameter is assumed to be constant. The solution of Einstein field equations has been obtained the best value for the Hubble parameter, energy density, isotropic pressure and cosmological term at present and at recombination. The physical and kinematical parameter of the model is also investigated.

**Key words:** FRW Model, Deceleration Parameter, Bulk Viscosity, Hubble parameter.

### 1. INTRODUCTION:

Friedmann [1] was first to obtain a general relativistic cosmological solution of Einstein's equation describing, expanding, spatially homogeneous and isotropic universe. He first recognized that the expansion starts in a super dense state viz. Big Bang and related these models to the redshift measurements (Hubble, [2]). Robertson and Walker in a series of papers (Robertson [3, 4] walker [5, 6]) proved that the line-element depicting spatially homogeneous and isotropic world models.

The distribution of matter can be satisfactorily described by a perfect fluid due to the large scale distribution of galaxies in our universe. However, observed physical phenomena such as the large entropy per baryon and the remarkable degree of isotropy of the cosmic microwave background radiation, suggest analysis of dissipative effects in cosmology. Furthermore, there are several processes which are expected to give rise to viscous effects. These are the decoupling of neutrinos during the radiation era and the decoupling of radiation and matter during the recombination era. Bulk viscosity is associated with the GUT phase transition and string creation. Misner [7–9] and Weinberg [10, 11] have studied the effect of viscosity on the evolution of cosmological models. Due to such assumption, dissipative processes are supposed to play a fundamental role in the evolution of the early Universe. The theory of relativistic dissipative fluids, created by Eckart [12] and Landau and Lifshitz [13] has many drawbacks, and it is known that it is incorrect in several aspects mainly those concerning causality and stability. Israel [14] formulates a new theory in order to solve these drawbacks. This theory was later developed by Israel and Stewart [15] into what is called transient or extended irreversible thermodynamics. The best currently available theory for analyzing dissipative processes in the Universe is the full causal thermodynamics developed by, Hiscock and Lindblom [16,17] and Hiscock and Salmonson [18]. The full causal bulk viscous thermodynamics has been extensively used to study the evolution of the early Universe and some astrophysical processes [19, 20]. However, due to the complicated nature of the evolution equations, very few exact cosmological solutions of the gravitational field equations are known in the framework of the full causal theory.

The paper paper, we start by reviewing the main components of a flat bulk viscous fluid FRW cosmological model  $\Lambda$ , and introduce the assumptions. These assumptions bring us to study to origin and evolution of universe. The model is found to be compatible with the results of recent cosmological observations.

### 2. Metric and Field Equation :

We consider homogeneous and isotropic spatially flat Rabertson-Walker line element of the form

$$ds^2 = -dt^2 + s^2(t)(dx^2 + dy^2 + dz^2) , \quad \dots (1)$$

where  $S(t)$  is the scale factor.

The energy-momentum tensor for bulk viscous fluid is taken as

$$T_{ij} = (\rho + p)v_i v_j + \bar{p}g_{ij} , \quad \dots (2)$$

where  $\rho$  is proper energy density and  $\bar{p}$  is the effective pressure given by



$$\bar{p} = p - \xi v_i^i \quad \dots (3)$$

satisfying equation of state.

In the above equation  $p$  is the isotropic pressure and  $v^i$  is the four-velocity vector satisfying  $v^i v_i = -1$ . The Einstein field equations (in gravitational units  $8 \pi G = c = 1$ ) and varying cosmological constant  $\Lambda(t)$ , in comoving system of coordinates to

$$\bar{p} - \Lambda = (2q - 1)H^2, \quad \dots (4)$$

$$\rho + \Lambda = 3H^2 \quad \dots (5)$$

In the above equation,  $H$  is the Hubble parameter and  $q$  is the deceleration parameter defined as

$$H = \frac{\dot{S}}{S}, \quad \dots (6)$$

$$q = -1 - \frac{\dot{H}}{H^2} = -\frac{S\ddot{S}}{\dot{S}^2} \quad \dots (7)$$

where an overhead dot ( $\dot{\phantom{x}}$ ) represents ordinary derivative with respect to  $t$ . The vanishing divergence of Einstein tensor gives rise to

$$\dot{\rho} + 3(\rho + \bar{p})H + \dot{\Lambda} = 0 \quad \dots (8)$$

### 3. Solution and Discussion:

The equations (4) & (5) are two equation involving with five unknown terms  $S$ ,  $\rho$ ,  $n$ ,  $p$  and  $\xi$ . Therefore, more equation connecting these variables.

To describe the dynamics of universe, Hubble parameter  $H$  and deceleration parameter  $q$  are important quantities. We choose

$$q = -\frac{S\ddot{S}}{\dot{S}^2} = n \quad \dots (9)$$

where  $n > 0$  is constant.

For this choice, we get scale factor  $S$ , expansion scalar  $\theta$ , spatial volume  $v$

$$S = (S_0 T)^{\frac{1}{1+n}} \quad \dots (10)$$

$$\theta = \frac{3}{(1+n)T} \quad \dots (11)$$

where  $S_0$  is constant of integration and set  $T = t_0 + t$ .

Secondly, we assume

$$\Lambda = 3\beta H^2 \quad \dots (12)$$

where  $\beta$  is constant [21, 22].

$$V = S^3 = (S_0 T)^{\frac{3}{1+n}} \quad \dots (13)$$

Next, we consider

$$\xi = \xi_0 \rho \quad \dots (14)$$

where  $\xi_0$  is constant [23].

From equations (11) & (1), we get

$$ds^2 = -dt^2 + (S_0 T)^{\frac{2}{1+n}} [dx^2 + dy^2 + dz^2] \quad \dots (15)$$

Matter density  $\rho$ , cosmological term  $\Lambda$ , isotropic pressure  $p$ , bulk viscosity  $\xi$  and the ratio  $\Omega = \Lambda / \rho$  for the model (15) are given by

$$\rho = \frac{3(1-\beta)}{(1+n)^2 T^2} \quad \dots (16)$$



$$\Lambda = \frac{3\beta}{(1+n)^2 T^2} \quad \dots (17)$$

$$p = \frac{1}{(1+n)^2 T^2} \left[ \frac{3(1-\beta)\xi_0}{(1+n)T} + (2n-1) \right] \quad \dots (18)$$

$$\xi = \frac{3\xi_0(1-\beta)}{(1+n)^2 T^2} \quad \dots (19)$$

$$\Omega = \frac{\beta}{1-\beta} \quad \dots (20)$$

We notice that the model start with big-bang at  $T = 0$ . The model of expansion decreases as time increases whereas spatial volume increases as time increases. At  $T = 0$ , the physical parameter  $\rho$ ,  $\Lambda$ ,  $p$  and  $\xi$  are infinite. As  $T = \infty$ ,  $\rho$ ,  $\Lambda$ ,  $p$  and  $\xi$  become zero. We notice that  $\Omega$  and  $\rho / \theta$  are constant throughout the evaluation of the universe.

#### 4. CONCLUSION:

In this paper, we have investigated FRW cosmological models with constant ratio of matter density and bulk viscosity, and constant deceleration parameter both. For  $n > 0$ , the model describes a decelerating universe throughout the evaluation of the universe. Let  $n = 0$ , we obtain  $H = T^{-1}$  and  $q = 0$ , so that galaxy moves with constant speed. The exact solution of Einstein field equation described for better understanding of origin and evolution of the universe.

#### REFERENCES:

1. Friedmann, A.: Z. (1922).*Physik*, **10**, 377.
2. Hubble, E.P. (1929).*Proc. Nat. Acad. Sci. (USA)*, **15**, 168.
3. Robertson, H.P. (1935).*Astrophys. J.*, **82**, 284.
4. Robertson, H.P. (1936).*Astrophys. J.*, **83**, 187, 257.
5. Walker, A.G. (1935).*Quart. J. Math. Oxford Ser.* **6**, 81.
6. Walker, A.G. (1936).*Proc. London Math. Soc. (2)*, **42**, 90.
7. C. W. Misner (1966).*Phys. Rev. Lett.* **19**, 533.
8. C. W. Misner (1967).*Nature*, **214**, 40.
9. C. W. Misner (1968).*Astrophys. J.* **151**, 431.
10. S. Weinberg (1971).*Astrophys. J.* **168**, 175.
11. S. Weinberg, Gravitation and Cosmology, (Wiley, New York 1972).
12. C. Eckart (1940).*Phys. Rev.* **58**, 919.
13. L. D. Landau and E. M. Lifshitz, Fluid Mechanics, (Oxford: Butterworth Heinemann 1987).
14. W. Israel (1976).*Ann. Phys. (NY)* **100**, 310.
15. W. Israel W and J. M. Stewart (1976).*Phys. Lett.* **A58**, 213.
16. W. A. Hiscock and L. Lindblom (1989).*Ann. Phys. (NY)* **151**, 466.
17. W. A. Hiscock and J. Salmonson (1991).*Phys. Rev.* **D43**, 3249.
18. W. A. Hiscock and L. Lindblom (1987).*Phys. Rev.* **D35**, 3723.
19. R. Maartens (1995).*Class. Quantum Grav.* **12**, 1455.
20. O. Cornejo-Pérez and J. A. Belinchón (2013).*Int. J. Mod. Phys.* **D22**, 1350031.
21. R.G. Vishwakarma (2001). *Class Quantum Grav.*, **18**, 1159.
22. J. C. Carvalho, J.A.S. Lima and I. Waga (1992). *Phys. Rev.*, **D46**, 2404.
23. R. Maartens (1995). *Class. Quantum Grav.*, **12**, 1455.