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Inverse and Forward Kinematics of 3R Planner Serial Robot

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Abstract: *Robots are replacing human work force to a huge extent. The robots are essential in the modern industries and manufacturing field for precise activities. The study of robotics is now an emerging filed in the research area. The robot has two types of as serial and parallel manipulator. In this work the serial manipulator of 3DOF planner robot has been conisdered. In the present paper, the kinematics modelling of a 3R planner robot has been done for the inverse and forward kinematics. The detailed mathematical modelling of these kinematics equations can help to derive the similar type of mathematical modelling for higher degree of freedom robot. The different set of output value of corresponding kinematics modelling have been tabulated in this work. The set of data available in the tables can be used for the predcition purpose.*

Key Words: *3-R Planner Robot, Serial Robot, Inverse kinematics, Forward kinematics.*

1. INTRODUCTION:

Robots are essential in human beings in every aspect of life. It is replacing human work force to a huge extent. Every kind of industrial applications like production, packaging, logistics etc use the application of robot. So the robotics is one of the most emerging area in the research filed. The study of the analysis and mathematical modelling of the robotics is important. Application of mechanical, electrical, electronics, computer science engineering is seen almost equivalently [1-2] in the study of robotics. The inverse kinematics modelling is helpful to find out the end effector position with help of each link length and movement [2-4]. The forward kinematics modelling supports to extract each link movement with help of the end effector position and each link length [1, 4-5]. The mathematical equations for the higher degree of freedom system is too much complicated that it is not possible to find out the mathematical equation by simple algebraic method. The DH matrix is essential for higher order system to find out the inverse and forward kinematics modelling [5-6]. These modelling of the serial manipulator can help the idea of the parallel manipulator [7-8] with higher degree of freedom [9-10]. The modern controllers [11-12] will be helpful to control the precise motion of these type of systems. The mathematical modelling will be helpful to get the prior knowledge before real-time experimental works. In this research work, the basic of the 3R planner serial robot has been discussed in the next section. The mathematical modelling of forward and inverse kinematics for 3R serial manipulator have been formulated step by step in the section 3. The different set of values extracted from the inverse and forward kinematics have been tabulated in the section 4. Theses set of values can be used in future for the prediction purpose by ANN methodology [13-15].

2. 3R planner robot:

3R Planner manipulator is called 3R because it has 3 revolving links. It has only rotary motion and no linear motions. It is called planner because its motion occurs only in 1 plane. This is a typical manipulator which is the precursors of all types of manipulators. The Fig. 1 depicts the 3R planner manipulator with link length l_1, l_2 and l_3 of link 1, 2 and 3 respectively and θ_1, θ_2 and θ_3 are angle of l_1 w.r.t X-axis, l_2 w.r.t l_1 and l_3 w.r.t to l_2 respectively. The end effector position have been taken here as X, Y and φ

$$\text{where } \varphi = \theta_1 + \theta_2 + \theta_3 \tag{2.1}$$

X and Y are the coordinate in x and y axis respectively.

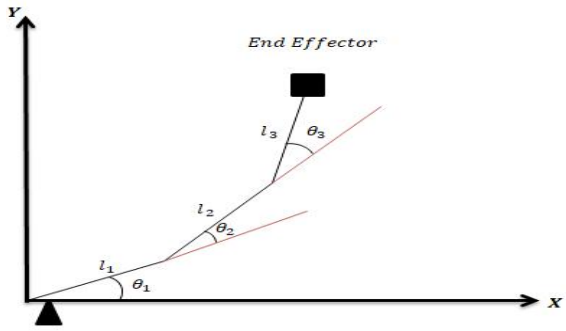

Figure 1. 3R serial planner manipulator.

3. Mathematical Modelling of Forward and Inverse Kinematics of 3R planner robot

Forward Kinematics

In forward kinematic analysis, the angular positions of the links θ_1, θ_2 and θ_3 are given and the position of the end effector (X,Y) have to be found out. The X & Y-coordinate of the end effector is basically the summation of the X & Y-coordinates of the individual links, hence

$$X = X_{l_1} + X_{l_2} + X_{l_3} \quad (2.2)$$

Now from the Fig.1 it can be written that

$$X_{l_1} = l_1 cos\theta_1 = l_1 C_1 \quad (2.3a)$$
$$X_{l_2} = l_2 \cos(\theta_1 + \theta_2) = l_2 C_{12} \quad (2.3b)$$
$$X_{l_3} = l_3 \cos(\theta_1 + \theta_2 + \theta_3) = l_3 C_{123} \quad (2.3c)$$

Similarly,

$$Y = Y_{l_1} + Y_{l_2} + Y_{l_3} \quad (2.4)$$
$$Y_{l_1} = l_1 sin\theta_1 = l_1 S_1 \quad (2.5a)$$
$$Y_{l_2} = l_2 \sin(\theta_1 + \theta_2) = l_2 S_{12} \quad (2.5b)$$
$$Y_{l_3} = l_3 \sin(\theta_1 + \theta_2 + \theta_3) = l_3 S_{123} \quad (2.5c)$$

So it can be written from (2.2, 2.3a-c) and (2.4, 2.5a-c

$$X = l_1 C_1 + l_2 C_{12} + l_3 C_{123} \quad (2.6a)$$

$$Y = l_1 S_1 + l_2 S_{12} + l_3 S_{123} \quad (2.6b)$$

The end effector position can be extracted by using (2.6a) and (2.6b) which are forward kinematics equations.

Inverse Kinematics:

In inverse kinematics, the position of the end effector (X, Y) are given and θ_1, θ_2 and θ_3 have to be found out. Now the forward kinematic equations (2.6a) and (2.6b) can be rewritten as

$$X - l_3 C_{123} = l_1 C_1 + l_2 C_{12} = m \quad (2.7a)$$
$$Y - l_3 S_{123} = l_1 S_1 + l_2 S_{12} = n \quad (2.7b)$$

From (2.7a) and (2.7b), it can also be written that

$$m^2 + n^2 = (l_1 C_1 + l_2 C_{12})^2 + (l_1 S_1 + l_2 S_{12})^2 \quad (2.8a)$$
$$= l_1^2 + l_2^2 + 2 l_1 C_1 l_2 C_{12} + 2 l_1 S_1 l_2 S_{12} \quad (2.8b)$$

$$[\because\ C_1^2 + S_1^2 = cos^2\theta_1 + sin^2\theta_2 = 1\ and\ C_{12}^2 + S_{12}^2 = cos^2(\theta_1 + \theta_2) + sin^2(\theta_1 + \theta_2) = 1] \quad (2.8c)$$

$$m^2 + n^2 = l_1^2 + l_2^2 + 2 l_1 l_2 (C_1 C_{12} + S_1 S_{12}) \quad (2.9a)$$
$$= l_1^2 + l_2^2 + 2 l_1 l_2 [cos\theta_1 \cos(\theta_1 + \theta_2) + sin\theta_1 \sin(\theta_1 + \theta_2)] \quad (2.9b)$$
$$= l_1^2 + l_2^2 + 2 l_1 l_2 \cos(\theta_1 - \theta_2 - \theta_1) \quad (2.9c)$$
$$= l_1^2 + l_2^2 + 2 l_1 l_2 cos\theta_2 \quad (2.9d)$$

Hence using (2.9d)

$$cos\theta_2 = \frac{m^2 + n^2 - l_1^2 - l_2^2}{2 l_1 l_2} \quad (2.10a)$$

$$\theta_2 = \cos^{-1}\frac{m^2+n^2-l_1^2-l_2^2}{2l_1l_2} \quad (2.10b)$$

Now, $$C_{12} = C_1C_2 + S_1S_2 \quad (2.11a)$$

So it can be written with help of (2.7a) and (2.11a) that

$$m = l_1C_1 + l_2(C_1C_2 + S_1S_2) \quad (2.11b)$$

$$= C_1(l_1 + l_2C_2) - l_2S_1S_2 \quad (2.11c)$$

Again, $$S_{12} = S_1C_2 + C_1S_2 \quad (2.12a)$$

So it can be written with help of (2.7b) and (2.12a) that

$$n = l_1S_1 + l_2(S_1C_2 + C_1S_2) \quad (2.12b)$$

$$= S_1(l_1 + l_2C_2) + l_2C_1S_2 \quad (2.12c)$$

Now, by $(2.11c) \times (l_1 + l_2C_2) + (2.12c) \times l_2S_2$

$$m(l_1 + l_2C_2) = C_1(l_1 + l_2C_2)^2 - l_2S_1S_2(l_1 + l_2C_2)$$

$$+ \quad nl_2S_2 = l_2S_1S_2(l_1 + l_2C_2) + C_1(l_2S_2)^2$$

$$m(l_1 + l_2C_2) + nl_2S_2 = C_1[(l_1 + l_2C_2)^2 + (l_2S_2)^2] \quad (2.13a)$$

So, $$C_1 = cos\theta_1 = \frac{m(l_1+l_2C_2)+nl_2S_2}{(l_1+l_2C_2)^2+(l_2S_2)^2} \quad (2.13b)$$

$$\theta_1 = \cos^{-1}\frac{m(l_1+l_2C_2)+nl_2S_2}{(l_1+l_2C_2)^2+(l_2S_2)^2} \quad (2.13c)$$

Hence using (2.1), (2.10b) and (2.13c) the value of θ_3 can be extracted as

$$\theta_3 = \varphi - \theta_1 - \theta_2 \quad (2.14)$$

The angular movement of each link length can be found out by using (2.10b), (2.13c) and (2.14) which are inverse kinematics equations.

4. RESULTS AND DISCUSSION:

4.1. Set of Data extracted from the forward kinematics mathematical modelling:

Now the data collecting from equation (2.6a) and (2.6b) for forward kinematics have been tabulated in Table1. In the Table 1, the given data are L_1, L_2, L_3 and $\theta_1, \theta_2, \theta_3$. The corresponding extracted values of X and Y have been tabulated with help of forward kinematic modelling established by (2.6a) and (2.6b).

Table:1 Set of data extracted from forward kinematics mathematical modelling

Sl No	L_1 (cm)	L_2 (cm)	L_3 (cm)	θ_1 (degree)	θ_2 (degree)	θ_3 (degree)	X (cm)	Y (cm)
1	8	5	3	5	4	3	15.84244	2.103153
2	8	5	3	10	8	6	15.37438	4.15448
3	8	5	3	15	12	9	14.60949	6.103861
4	8	5	3	20	16	12	13.57002	7.904522
5	8	5	3	25	20	15	12.28600	9.514556
6	8	5	3	30	24	18	10.79418	10.89825
7	8	5	3	35	28	21	9.136754	12.02721
8	8	5	3	40	32	24	7.359855	12.88115
9	8	5	3	45	36	27	5.511976	13.44847
10	8	5	3	50	40	30	3.642301	13.72643
11	8	5	3	55	44	33	1.799047	13.72109
12	10	8	6	60	48	36	-2.326240	19.79542
13	10	8	6	65	52	39	-4.887010	18.63155
14	10	8	6	70	56	42	-7.150970	17.11653
15	10	8	6	75	60	45	-9.06866	15.31611
16	10	8	6	80	64	48	-10.6045	13.30289
17	10	8	6	85	68	51	-11.7378	11.15345

18	10	8	6	90	72	54	-12.4626	8.945424
19	10	8	6	95	76	57	-12.7878	6.754554
20	10	8	6	100	80	60	-12.7365	4.651925
21	10	8	6	105	84	63	-12.3438	2.701443
22	10	8	6	110	88	66	-11.6558	0.957659
23	15	10	8	115	92	69	-14.4131	1.098537
24	15	10	8	120	96	72	-13.118	-0.49592
25	15	10	8	125	100	75	-11.6747	-1.71199
26	15	10	8	130	104	78	-10.1666	-2.54466
27	15	10	8	135	108	81	-8.67437	-3.00575
28	15	10	8	140	112	84	-7.27247	-3.12264
29	15	10	8	145	116	87	-6.02644	-2.93653
30	15	10	8	150	120	90	-4.99038	-2.5000
31	15	10	8	155	124	93	-4.20509	-1.87432
32	15	10	8	160	128	96	-3.69686	-1.12637
33	15	10	8	165	132	99	-3.47685	-0.3255
34	18	15	12	170	136	102	-0.88019	-0.09185
35	18	15	12	175	140	105	-1.3249	1.354507
36	18	15	12	180	144	108	-2.15654	2.595899
37	18	15	12	185	148	111	-3.31207	3.555602
38	18	15	12	190	152	114	-4.71503	4.173341
39	18	15	12	195	156	117	-6.27954	4.407418
40	18	15	12	200	160	120	-7.91447	4.235942
41	25	20	19	185	148	111	-5.098696	7.637212
42	25	20	19	190	152	114	-7.585104	8.3743716
43	25	20	19	195	156	117	-10.26570	8.470908
44	25	20	19	200	160	120	-12.99231	7.9039790
45	25	20	19	205	164	123	-15.61740	6.6829844
46	27	25	21	210	168	126	-16.59562	6.5689151
47	27	25	21	215	172	129	-19.0263	4.4046682
48	27	25	21	220	176	132	-20.99887	1.7055113
49	27	25	21	225	180	135	-22.41421	-1.4142135
50	27	25	21	230	184	138	-23.20173	-4.8239206

4.2. Set of Data extracted from the inverse kinematics mathematical modelling:

Now the data collecting from equations (2.10b), (2.13c) and (2.14) of inverse kinematics have been tabulated in Table2. In the Table 2, the given data are L_1, L_2, L_3 and X and Y , the corresponding extracted value of θ_1, θ_2 and θ_3 have been tabulated with help of inverse kinematic modelling established in (2.10b), (2.13c) and (2.14).

Table:2 Set of data extracted from inverse kinematics mathematical modelling

Sl No.	X (cm)	Y (cm)	L_1 (cm)	L_2 (cm)	L3 (cm)	θ_1 (degree)	θ_2 (degree)	θ_3 (degree)
1	15.84244	2.103153	8	5	3	5	4	3
2	15.37438	4.15448	8	5	3	10	8	6
3	14.60949	6.103861	8	5	3	15	12	9

4	13.57002	7.904522	8	5	3	20	16	12
5	12.286	9.514556	8	5	3	25	20	15
6	10.79418	10.89825	8	5	3	30	24	18
7	9.136754	12.02721	8	5	3	35	28	21
8	7.359855	12.88115	8	5	3	40	32	24
9	5.511976	13.44847	8	5	3	45	36	27
10	3.642301	13.72643	8	5	3	50	40	30
11	1.799047	13.72109	8	5	3	55	44	33
12	-2.32624	19.79542	10	8	6	60	48	36
13	-4.88701	18.63155	10	8	6	65	52	39
14	-7.15097	17.11653	10	8	6	70	56	42
15	-9.06866	15.31611	10	8	6	75	60	45
16	-10.6045	13.30289	10	8	6	80	64	48
17	-11.7378	11.15345	10	8	6	85	68	51
18	-12.4626	8.945424	10	8	6	90	72	54
19	-12.7878	6.754554	10	8	6	95	76	57
20	-12.7365	4.651925	10	8	6	100	80	60
21	-12.3438	2.701443	10	8	6	105	84	63
22	-11.6558	0.957659	10	8	6	110	88	66
23	-14.4131	1.098537	15	10	8	115	92	69
24	-13.118	-0.49592	15	10	8	120	96	72
25	-11.6747	-1.71199	15	10	8	125	100	75
26	-10.1666	-2.54466	15	10	8	130	104	78
27	-8.67437	-3.00575	15	10	8	135	108	81
28	-7.27247	-3.12264	15	10	8	140	112	84
29	-6.02644	-2.93653	15	10	8	145	116	87
30	-4.99038	-2.5000	15	10	8	150	120	90
31	-4.20509	-1.87432	15	10	8	155	124	93
32	-3.69686	-1.12637	15	10	8	160	128	96
33	-3.47685	-0.3255	15	10	8	165	132	99
34	-0.88019	-0.09185	18	15	12	170	136	102
35	-1.3249	1.354507	18	15	12	175	140	105
36	-2.15654	2.595899	18	15	12	180	144	108
37	-3.31207	3.555602	18	15	12	185	148	111
38	-4.71503	4.173341	18	15	12	190	152	114
39	-6.27954	4.407418	18	15	12	195	156	117
40	-7.91447	4.235942	18	15	12	200	160	120
41	0.874346	-26.9842	25	20	19	280	224	168
42	4.021669	-26.2363	25	20	19	285	228	171
43	6.886737	-25.04	25	20	19	290	232	174
44	9.396494	-23.4793	25	20	19	295	236	177
45	11.5000	-21.6506	25	20	19	300	240	180
46	11.33545	-21.6618	27	25	21	305	244	183
47	12.76331	-19.8672	27	25	21	310	248	186
48	13.80608	-18.0982	27	25	21	315	252	189

49	14.50952	-16.4439	27	25	21	320	256	192
50	14.93944	-14.9777	27	25	21	325	260	195

Theses set of values in Tables 1 and 2 can be used in future for the prediction purpose by ANN methodology [13-15].

5. CONCLUSION:

The inverse and forward kinematics mathematical modelling of 3R serial manipulator have been formulated in this present study. The mathematical expressions help to find out the corresponding output values which have been tabulated in this paper work. The tabulated data set will help to predict the kinematics modelling of 3R robot. The testing and validation data of the prediction process can be taken from the tables. The prediction process ANN can be used in further for the modelling of higher degree of freedom robotic system.

REFERENCES :

1 T. P. Singh, P. Suresh, S. Chandan, Forward and Inverse Kinematic Analysis of Robotic Manipulators, IRJET, Vol. 04 (2017), pp. 459-1469.
2 A.Afloare, N. Apostolescu, A. Chira, C.E. Munteanu, Inverse Kinematic solution of a 6 DOF serial manipulator, Proceedings of the 30th DAAAM International Symposium, Vienna, Austria, (2019), pp.0628-0631.
3 Z. Lu, C. Xu, Q. Pan, X. Zhao, X. Li, Inverse Kinematic Analysis and Evaluation of a Robot for Nondestructive Testing Application, Hindawi Publishing Corporation Journal of Robotics, (2015), pp. 1-7.
4 X. De, C.A.A. Calderon, J.Q. Gan, Hu. H, An Analysis of the Inverse Kinematics for a 5- DOF Manipulator, International Journal of Automation and Computing, Vol. 2 (2005), pp. 114-124.
5 E. M. Rosales, J. Q. Gan, Forward and inverse kinematics models for a 5-DOF pioneer 2 robot arm, Technical report, University of Essex, 2003.
6 J.Q. Gan, E. Oyama, E.M. Rosales, H. Hu, A complete analytical solution to the inverse kinematics of the Pioneer 2 robotic arm, Robotica, Cambridge University Press. Vol. 23(2005), pp. 123–129.
7 S. Dasmahapatra, M. Ghosh, Workspace Identification of Stewart Platform, International Journal of Engineering and Advanced Technology, volume 9, issue-3 (2020), pp. 1903-1907.
8 M. Ghosh, S. Dasmahapatra, Kinematic Modeling of Stewart Platform, Springer Nature Switzerland AG, S. Dawn et al. (Eds.): ICIMSAT 2019, LAIS12 (2020), pp. 693-701.
9 S. Dasmahapatra, D. Saha, R. Saha, D. Sanyal, D. Lahiri, J. P. Singh, Analysis of 6-DOF motion with PI controller in electrohydraulic Stewart platform. IEEE 1st CMI (2016), pp. 186-190.
10 S. H. Mullick, S. Dasmahapatra, Combined Motion Generation by Electro-Hydraulic Stewart Platform for Manufacturing Industries, Springer Nature Switzerland AG 2020, S. Dawn et al. (Eds.): ICIMSAT 2019, LAIS 12, pp. 596–604.
11 S. Dasmahapatra, B.K. Sarkar, R. Saha, A. Chatterjee, S. Mookherjee, D. Sanyal, Design of an adaptive-fuzzy-bias-SMC and validation for a rugged electrohydraulic system. IEEE/ASME Trans. Mechatronics, Vol. 20, No. 6 (2015), pp. 2708-2715.
12 S. Dasmahapatra, R. Saha, S. Mookherjee, D. Sanyal, Designing an input-linearized adaptive sliding mode coupled nonlinear integral controller, IEEE/ASME Trans. Mechatronics, Vol. 23, No. 6 (2018), pp. 2888-2895.
13 B. Daya, S. Khawandi, M. Akoum, Applying Neural Network Architecture for Inverse Kinematics Problem in Robotics, J. Software Engineering & Applications, Vol. 3, (2010), pp. 230-239.
14 L. Wei, H. Wang, Y. Li , A new solution for inverse kinematics of manipulator based on neural network, Machine Learning and Cybernetics, Vol. 2, 2003 pp. 1201–1203.
15 J. Guo, V. Cherkassky, A solution to the inverse kinematic problem in robotics using neural network processing, International Joint Conference on Neural Networks, Vol. 2, 1989, pp. 299–304.