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SQUARE AND SAW TOOTH MOTIONS OF PARALLEL MANIPULATOR - STEWART PLATFORM

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Abstract: *Square and Saw tooth motions of 6-DOF Parallel Manipulator-Stewart Platform have been executed and studied in this paper. Top movable platform of the Manipulator is coupled with its fixed base frame by six linear actuators. Individual movement of each actuator for a particular pose of the movable platform has been formulated from the mathematical modelling of inverse kinematics. Different linear motions like surge, sway and heave, and rotational motions like roll, pitch and yaw illustrates different poses of the movable platform. The square and saw tooth type of different linear and rotational motions can be implemented in automobile industry, painting, welding and precise robotic surgery. Corresponding actuator-piston lengths for pose demand of each basic linear motion and rotational motion have been extracted through inverse kinematics modelling. The motions of the piston have been plotted against the elapsed time to find out the range of each motion.*

Key Words: *Parallel Manipulator, Inverse Kinematics, Square Motions, Saw tooth Motions.*

1. INTRODUCTION:

Six degrees-of-freedom parallel manipulator-Stewart platform has a movable frame to maneuver under heavy load with three translational and three angular motions along fundamental axes. Six linear electrohydraulic actuators [1] connect top movable platform with the fixed base frame resulting in a parallel structure. The parallel architecture provides higher stiffness, rigidity [2] and high ratio of payload to mass because of distribution of load among all serial chains between fixed and moving platform. Stewart platform is popularly used as motion simulator [3] of an airplane or ship by generating definite motions for training purposes within laboratory scale. High precision parallel manipulator can be used to perform medical surgery [4] and also to resolve the stabilization problem of a radio-telescope [5, 6]. Modern industrial automation performances like Laser cutting, welding, painting [7, 8] by parallel manipulator creates motivating research trends. Challenging research works make parallel manipulator [9] as the best choice for industrial application due to their high speed and precision level compared to the serial manipulator [10].

Kinematic modelling transforms mathematical modelling of parallel manipulator for motion simulation of moving platform as well as motion of each actuator [11, 12]. Inverse kinematic method provides [13, 14] the calculation of actuator responses for various pose demands like surge, sway, heave, roll, pitch and yaw. Precise control is incorporated with individual motion of the actuators in real time applications [15-17]. Present research work deals with the study of three translational motions of the moving platform as surge, sway, heave and three rotational motions as roll, pitch, yaw for square and triangular wave type along with corresponding individual actuator-piston movements. The brief description of the parallel manipulator have been explained in the next section followed by motion simulation modelling using inverse kinematics. Then motion simulations of the platform pose and the corresponding actuator responses have been illustrated and discussed in view of different manufacturing applications.

2. Parallel Manipulator Description:

Parallel manipulator-Stewart platform with six degrees of freedom pose demands for a payload depicted in figure 1 has been referred from Mintu et al. (2020). It has a planer moving platform **M** at the top and a semi-regular hexagonal fixed base frame **F** connected together with six actuators. The top joint, cylinder and bottom joint are denoted respectively as

Ti, *Ci* and *Bi* using the letter *i* to identify one of the actuators 1 to 6. The horizontal plane *OXY* of the stationary global system *OXYZ*, contains all the bottom points of actuators. The orientation of the moving platform is described by the moving coordinate system *opqr* with the initial vertical height *h* between the fixed origin *O* and moving origin *o*. The position vector of the point *o* in global coordinate system can be framed as

$$\mathbf{d} + h\hat{k} = (x_o \quad y_o \quad z_o + h)^T \tag{1}$$

The translational motions of **M** from the initial position *o* can be formulated as

$$\mathbf{d} = \mathbf{x}_o^T \hat{\mathbf{e}}_O , \tag{2}$$

where the translation vector and the unit vectors are respectively given by

$$\mathbf{x}_o = (x_o \quad y_o \quad z_o)^T , \tag{3}$$

$$\hat{\mathbf{e}}_O = (\hat{i} \quad \hat{j} \quad \hat{k})^T . \tag{4}$$

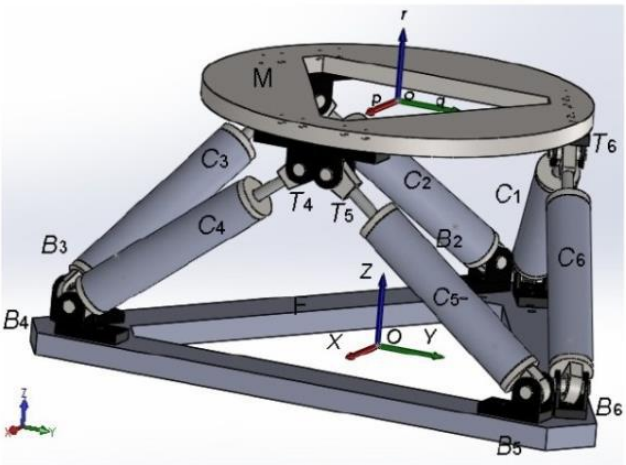

Figure 1 Parallel Manipulator- Stewart Platform with six actuators. [Source: Mintu et al. (2020)]

For the corresponding angular displacements α, β and γ about *p*, *q* and *r* axes respectively called roll, pitch and yaw describe the orientation of the platform as a vector

$$\boldsymbol{\theta} = (\alpha \ \beta \ \gamma)^T . \tag{5}$$

The pose of movable platform motion can be formulated using (2) and (5) as

$$\mathbf{q} = (\mathbf{d}^T \ \boldsymbol{\theta}^T)^T = (x_o \quad y_o \quad z_o \quad \alpha \quad \beta \quad \gamma)^T . \tag{6}$$

3. Mathematical Modelling of Inverse Kinematics for Parallel Manipulator:

Piston actuation vector for six actuators are defined by the position of center of the top joint relative to center of the bottom joint as

$$\mathbf{l}_i = \hat{\mathbf{e}}_O^T \mathbf{x}_i = \hat{\mathbf{e}}_O^T (\mathbf{R}^T \mathbf{p}_{Ti} + \mathbf{d} - \mathbf{x}_{Bi}) , \tag{7}$$

Where leg length vector $\mathbf{x}_i = (x_{Ti} - x_{Bi} \ \ y_{Ti} - y_{Bi} \ \ z_{Ti} - z_{Bi})^T \hat{\mathbf{e}}_O$, (8)

position vector of *Bi* $\mathbf{x}_{Bi} = (x_{Bi} \ \ y_{Bi} \ \ z_{Bi})^T \hat{\mathbf{e}}_O$, (9)

position vector of *Ti* $\mathbf{x}_{Ti} = (x_{Ti} \ \ y_{Ti} \ \ z_{Ti})^T \hat{\mathbf{e}}_O$, (10)

position vector of *Ti* in the moving coordinate system $\mathbf{p}_{Ti} = (p_{Ti} \quad q_{Ti} \quad r_{Ti}) \hat{\mathbf{e}}_o$, (11)

and the rotational matrix **R** [18] to transform a vector from moving to stationary coordinate system is defined as

$$\mathbf{R} = \mathbf{R}_\alpha \mathbf{R}_\beta \mathbf{R}_\gamma , \tag{12}$$

The rotation by α about p axis, β about q axis and γ about r axis are given respectively as

$$\mathbf{R}_{\alpha}=\begin{bmatrix}1 & 0 & 0\\ 0 & c\alpha & s\alpha\\ 0 & -s\alpha & c\alpha\end{bmatrix}, \tag{13}$$

$$\mathbf{R}_{\beta}=\begin{bmatrix}c\beta & 0 & -s\beta\\ 0 & 1 & 0\\ s\beta & 0 & c\beta\end{bmatrix}, \tag{14}$$

and $$\mathbf{R}_{\gamma}=\begin{bmatrix}c\gamma & s\gamma & 0\\ -s\gamma & c\gamma & 0\\ 0 & 0 & 1\end{bmatrix}. \tag{15}$$

The mathematical expression mentioned in (7) is used in this paper to find out the corresponding piston motion for the respective platform pose. The platform pose is user defined value of translation motion **d** as surge (x), sway (y), heave (z) and rotational motion $\boldsymbol{\theta}$ as roll (α), pitch (β), yaw (γ). The pose of the platform may be sinusoidal type, square type, saw tooth type or trapezoidal type of different basic translation or rotational motion or their different combination. In this work, square and saw tooth motions for basic translation and rotation motions have been studied in the next section.

4. Results and Discussion:

Each actuator-piston motion corresponding to different types of platform movements has been illuatrated in this section. The response of actuators for square wave and saw tooth type of motion of the parallel manipulator has been achived in Matlab using inverse kinematics (7).

The square wave and saw-tooth demand of the moving platform has been formulated respectively as

$$\text{Square wave pose demand} = a.\ \text{square}\ (2.\ pi.\ f.\ t) \tag{16}$$

$$\text{Saw-tooth pose demand} = a.\ \text{sawtooth}\ (2.\ pi.\ f.\ t) \tag{17}$$

where amplitude a, frequency f and time t. The square type of motion in (16) is the combination of two step motions whereas the saw tooth motion in (17) is the combination of two alternative linear motions.

The main objective of the present research work is to analyse the response of individual actuator for three basic linear motions along with three basic angular motions of the top movable platform with square wave and saw-tooth type demand. Each of the three basic pure linear motions Heave, Surge and Sway are illustrated in figures 2 to 4 for square wave and saw tooth pose demand. The maximum span of length of each actuator-piston has been adopted from Dasmahapatra et al. (2016) as 0 to 0.15m. The maximum achievable amplitude of the motion of moving platform for square and saw-tooth demand during Heave, Surge and Sway has been identified as 0.114 m, 0.1m and 0.09 m respectively with the frequency of 0.5 Hz keeping actuator-piston movement within the span of length of the actuator-piston. These results provides the knowledge about the working range of the moving platform prior to the design of the manipulator.

Motion simulations in figures 2 to 4 indicate that the same movement of each actuator-piston for Heave motion but different displacements for Surge and Sway. However there are actuator pairs 1-2, 3-6 and 4-5 for Surge and actuator pair 4-5 has undergone much lower displacement due to the actuator arrangement for this particular cofiguration of the parallel manipulator. Actuator pairs 1-3, 2-6 exist for Sway, when the motion of actuators 4 and 5 are dissimilar from each other. These observations provide information regarding the nature of actuator-piston movements which is important before real time experimentation.

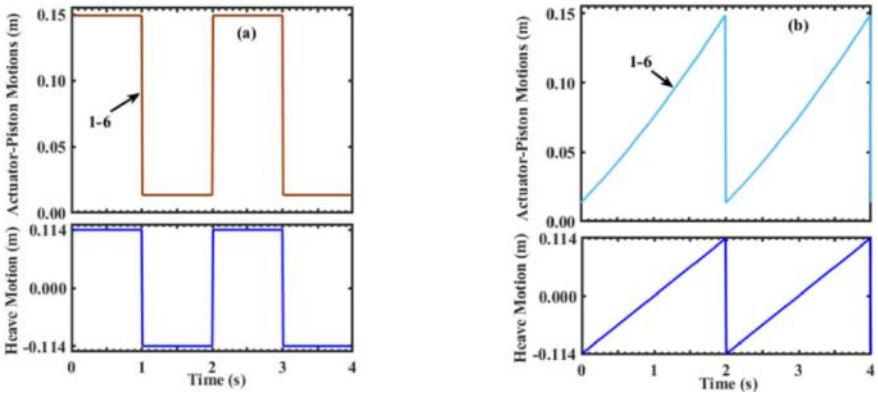

Figure 2 Actuator motions for 0.114 m and 0.5 Hz heave motion of the parallel manipulator with (a) square wave and (b) saw- tooth.

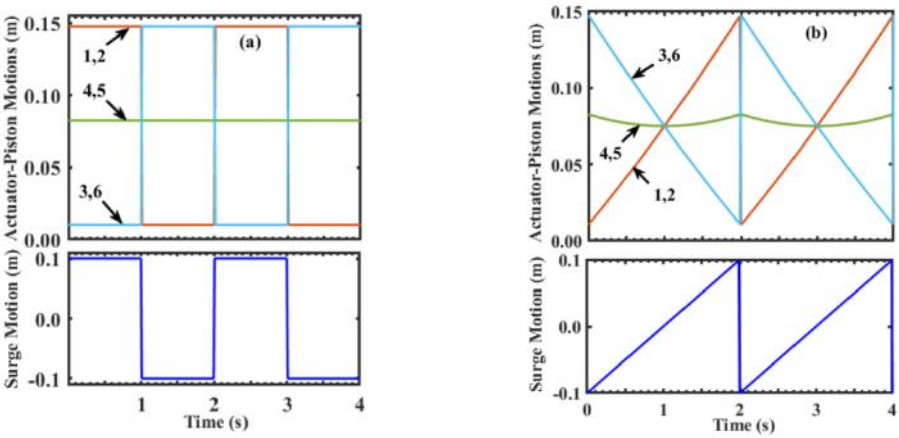

Figure 3 Actuator motions for 0.1 m and 0.5 Hz surge motion of the parallel manipulator with (a) square wave and (b) saw-tooth.

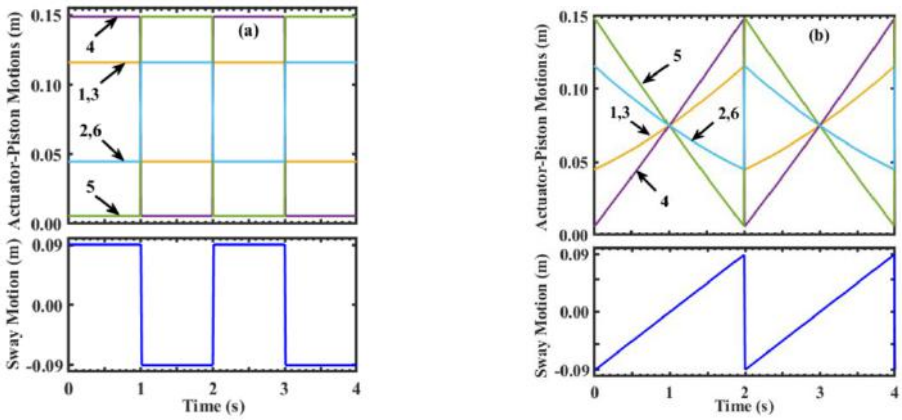

Figure 4 Actuator motions for 0.09 m and 0.5 Hz sway motion of the parallel manipulator with (a) square wave and (b) saw-tooth.

Each of the three basic pure rotational motions Roll, Pitch and Yaw are illustrated in figures 5 to 7 for square wave and saw tooth pose demand. The permissible amplitude of the motion of moving platform for square and saw-tooth demand during Roll, Pitch and Yaw has been identified as 19°, 17° and 13° respectively with the frequency of 0.5 Hz within the span of length of the actuator-piston. Motion simulation with square and saw-tooth type demand is the prerequisite for the precise path tracing during real time applications like laser cutting, welding etc. Simulation works as in figure 5 to 7 disclose that dissimilar movement of each actuator-piston for Roll motion as well as different displacements for Pitch and Yaw. However actuator pairs 1-2, 3-6 and 4-5 exist for Pitch whereas the Yaw shows different movements from other angular motions.The motions of two pairs of three legs 1-3-5 and 2-4-6 are out-of-phase by 180^0 for Yaw demand.

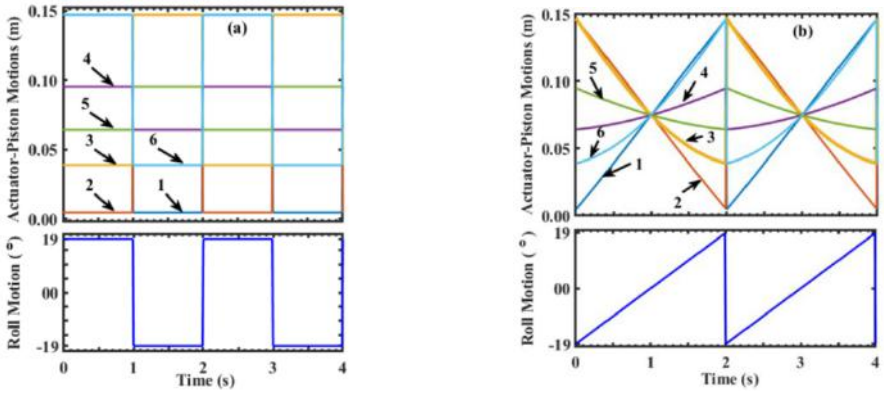

Figure 5 Actuator motions for 19° and 0.5 Hz roll motion of the parallel manipulator with (a) square wave and (b) saw-tooth.

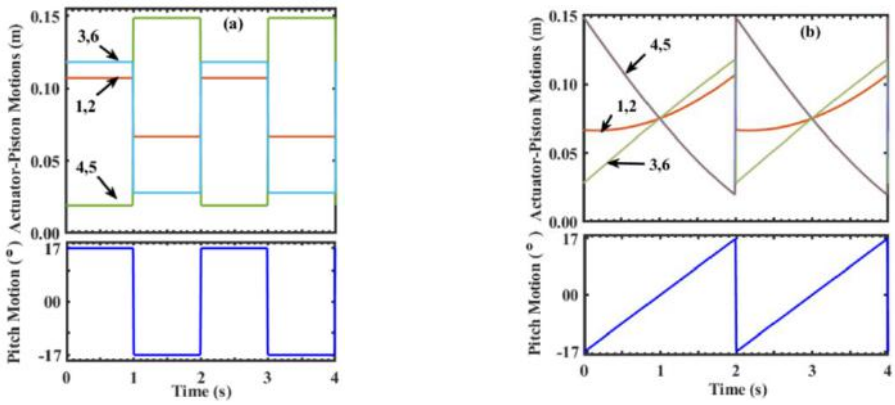

Figure 6 Actuator motions for 17° and 0.5 Hz pitch motion of the parallel manipulator with (a) square wave and (b) saw-tooth.

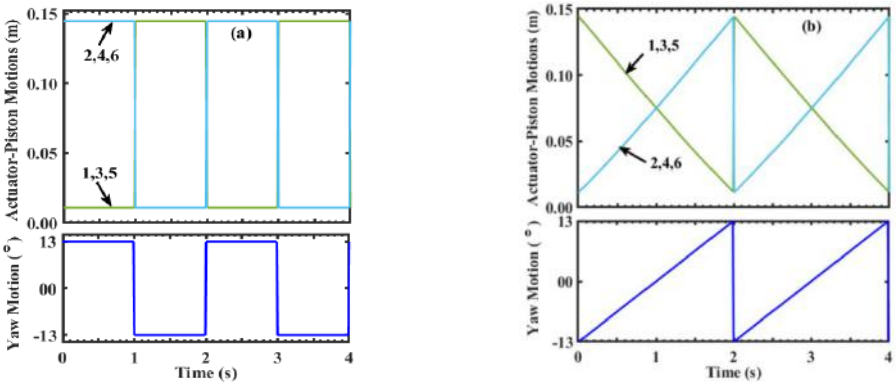

Figure 7 Actuator motions for 13° and 0.5 Hz yaw motion of the parallel manipulator with (a) square wave and (b) saw-tooth.

The square type of motions in figures 2(a), 3(a), 4(a), 5(a), 6(a) and 7(a) can be applied in manufacturing applications as welding techniques. The welding can be done in two different positions for a given time period. These type of motions can also be implemented in different robotic surgery operations. The corresponding saw tooth motions in figures 2(b), 3(b), 4(b), 5(b), 6(b) and 7(b) can also be implemented in automobile industry, painting and robotic surgery. Two alternative linear motions in saw tooth motion can be useful for precise surgery by Stewart Platform.

Singh et al. (2016) have done the experimental works of Stewart Platform for sinusoidal type of basic linear and rotational motions to find out the range of each actuator-piston displacement. In this present work the maximum achievable amplitude range of the moving platform for different linear and rotational motions for square and saw tooth type demand has been presented in Tables 1 and 2. The corresponding actuator-piston displacement has been extracted in Matlab environment by using Inverse Kinematics mathematical modelling (7) for the parallel manipulator. It can be revealed from the real time work of Sing et al. (2016) and the present study in Tables 1 and 2 that similar type of pattern in the motion of each actuator-piston for maximum and minimum platform demand.

Table 1 Maximum and Minimum displacements of actuator-piston for three pure linear motions of square and sawtooth demand

Pose Demand	Maximum Amplitude (m)	Maximum and Minimum Actuator-piston Response (m)					
		1	2	3	4	5	6
Heave – Square	**0.114**	**0.1494, 0.0135**	**0.1494, 0.0135**	**0.1494, 0.0135**	**0.1494, 0.0135**	**0.1494, 0.0135**	**0.1494, 0.0135**
Heave – Saw-tooth	**0.114**	**0.1486, 0.0135**	**0.1486, 0.0135**	**0.1486, 0.0135**	**0.1486, 0.0135**	**0.1486, 0.0135**	**0.1486, 0.0135**
Surge – Square	**0.100**	**0.1478, 0.0104**	**0.1478, 0.0104**	**0.1478, 0.0104**	0.0827, 0.0827	0.0827, 0.0827	**0.1478, 0.0104**
Surge – Saw-tooth	**0.100**	**0.1470, 0.0109**	**0.1470, 0.0109**	**0.1470, 0.0109**	0.0825, 0.0750	0.0825, 0.0750	**0.1470, 0.0109**
Sway – Square	**0.090**	0.1159, 0.0446	0.1159, 0.0446	0.1159, 0.0446	**0.1489, 0.0057**	**0.1489, 0.0057**	0.1159, 0.0446
Sway – Saw-tooth	**0.090**	0.1155, 0.0449	0.1155, 0.0449	0.1155, 0.0449	**0.1489, 0.0064**	**0.1489, 0.0064**	0.1155, 0.0449

Table 2 Maximum and Minimum displacements of actuator-piston for three pure rotational motions square and sawtooth demand

Pose Demand	Maximum Amplitude (degree)	Maximum and Minimum Actuator-piston Response (m)					
		1	2	3	4	5	6
Roll – Square	**19**	**0.1469, 0.0046**	**0.1469, 0.0046**	**0.1471, 0.0389**	0.0952, 0.0642	0.0952, 0.0642	**0.1469 0.0389**
Roll – Saw-tooth	**19**	**0.1462, 0.0046**	**0.1462, 0.0046**	**0.1471, 0.0390**	0.0947, 0.0642	0.0952, 0.0642	**0.1454, 0.0390**
Pitch – Square	**17**	0.1073, 0.0666	0.1073, 0.0666	0.1182, 0.0279	**0.1487, 0.0189**	**0.1487, 0.0189**	0.1182, 0.0279
Pitch – Saw-tooth	**17**	0.1056, 0.0666	0.1056, 0.0666	0.1178, 0.0279	**0.1487, 0.0203**	**0.1487, 0.0203**	0.1178, 0.0279
Yaw – Square	**13**	**0.1446, 0.0110**	**0.1446, 0.0110**	**0.1446, 0.0110**	**0.1446, 0.0110**	**0.1446, 0.0110**	**0.1446, 0.0110**
Yaw – Saw-tooth	**13**	**0.1446, 0.0110**	**0.1439, 0.0110**	**0.1446, 0.0110**	**0.1439, 0.0110**	**0.1446, 0.0110**	**0.1439, 0.0110**

5. Conclusion:

Motion simulation with square wave and saw-tooth motion by means of the mathematical modelling of inverse kinematics delivers the information about the maneuvering of the moving platform of the manipulator. The maximum possible amplitude for the moving platform have been identified satisfying feasibility criteria by keeping actuator-piston movement within the span of 0.15 m. Thus the present simulation work with detailed information about the workspace of the moving platform encourages real time experimental work on Parallel Manipulator. The basic linear and rotational motions with square and saw-tooth demand are found their practicality in motion generation in modern advanced

manufacturing industry and other several operations as modern robotic surgery. Some suitable advanced controllers can be designed for specific path tracing with controlled motion in real time applications.

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