# Inverse and Forward Kinematics of 2R Planner Serial Robot 

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#### Abstract

The robots are essential in the modern industries and manufacturing field for precise activities. Robots are replacing human work force to a huge extent. The study of robotics is now an emerging filed in the research area. The robot has two types of as serial and parallel manipulator. In this work the serial manipulator of 2DOF planner robot has been conisdered. In the present paper, the kinematics modelling of a $2 R$ planner robot has been done for the inverse and forward kinematics. The detailed mathematical modelling of these kinematics equations can help to derive the similar type of mathematical modelling for higher degree of freedom robot. The different set of output value of corresponding kinematics modelling have been tabulated in this work.


Key Words: 2-R Planner Robot, Serial Robot, Inverse kinematics, Forward kinematics

## 1. INTRODUCTION:

Now a days robots are seen to be replacing the human interactions in almost every sphere of life. In almost every kind of industrial applications like production, packaging, logistics etc. Robots are replacing human work force to a huge extent. Hence the study of robot and their kinematics have emerged out to be a very important subject in the field of mechanical engineering and mathematical analysis. In the study of robotics, application of mechanical, electrical, electronics, computer science engineering is seen almost equivalently [1-2]. The mathematical modelling of inverse kinematics of the robotics is required to find out the end effector position with help of each link length and movement [2-4]. The mathematical modelling of forward kinematics of the robotics is essential to extract each link movement with help of the end effector position and each link length [1, 4-5]. But apart from this studies a huge ocean of mathematical applications are also seen in this field such as DH Matrix analysis for the inverse and forward kinematic equations of robot or the complicated algebraic analysis for the analysis of the same [5-6]. These type of kinematics equations can be helpful for the idea of the parallel manipulators [7-9]. The control of the motion of these manipulator can be done with help of modern controllers [10-11]. The mathematical modelling can be implemented to get the prior knowledge before real-time experimental works. In this research work, the basic of the 2-R planner serial robot has been discussed in the next section. The mathematical modelling of forward and inverse kinematics for 2 R serial manipulator have been formulated step by step in the next section 3 . The different set of values extracted from the inverse and forward kinematics have been tabulated in the section 4. Theses set of values can be used in future for the prediction purpose by ANN methodology [12].

## 2. 2R PLANNER ROBOT:

2R Planner manipulator is called 2R because it has 2 revolving links. It has only rotary motion and no linear motions. It is called planner because its motion occurs only in 1 plane. This is a typical manipulator which is the precursors of all types of manipulators.


Figure 1. 2R serial planner manipulator.

The Fig. 1 depicts the 2 R planner manipulator with link length $l_{1}$, and $l_{2}$ of link 1 and 2 respectively and $\theta_{1}$ and $\theta_{2}$ are angle of $l_{1}$ w.r.t X -axis and $l_{2}$ w.r.t $l_{1}$ respectively. The end effector position have been taken here as $\mathrm{X}, \mathrm{Y}$ and $\varphi$

$$
\begin{equation*}
\text { where } \varphi=\theta_{1}+\theta_{2} \tag{2.1}
\end{equation*}
$$

X and Y are the coordinate in x and y axis respectively.

### 2.1. Mathematical Modelling of Forward and Inverse Kinematics of 2R planner robot Forward Kinematics

In forward kinematic analysis the angular positions of the links $\theta_{1}$ and $\theta_{2}$ are given and the position of the end effector ( $\mathrm{X}, \mathrm{Y}$ ) have to be found out. The X \& Y -coordinate of the end effector is basically the summation of the X \& Y-coordinates of the individual links, hence

$$
\begin{equation*}
X=X_{l_{1}}+X_{l_{2}} \tag{2.2}
\end{equation*}
$$

Now from the Fig. 1 it can be written that

$$
\begin{align*}
& X_{l_{1}}=l_{1} \cos \theta_{1}=l_{1} C_{1}  \tag{2.3a}\\
& \quad X_{l_{2}}=l_{2} \cos \left(\theta_{1}+\theta_{2}\right)=l_{2} C_{12} \tag{2.3b}
\end{align*}
$$

Similarly,

$$
\begin{align*}
& Y=Y_{l_{1}}+Y_{l_{2}}  \tag{2.4}\\
& \quad Y_{l_{1}}=l_{1} \sin \theta_{1}=l_{1} S_{1}  \tag{2.5a}\\
& \quad Y_{l_{2}}=l_{2} \sin \left(\theta_{1}+\theta_{2}\right)=l_{2} S_{12} \tag{2.5a}
\end{align*}
$$

So it can be written from (2.2, 2.3a-b) and (2.4, 2.5a-b)

$$
\begin{align*}
& X=l_{1} C_{1}+l_{2} C_{12}  \tag{2.6a}\\
& Y=l_{1} S_{1}+l_{2} S_{12} \tag{2.6b}
\end{align*}
$$

The end effector position can be extracted by using (2.6a) and (2.6b) which are forward kinematics equations.

## Inverse Kinematics:

In inverse kinematics the position of the end effector $(\mathrm{X}, \mathrm{Y})$ are given and $\theta_{1}$ and $\theta_{2}$ have to be found out. From (2.6a) and (2.6b) it can be written that

$$
\begin{align*}
X^{2}+Y^{2} & =\left(l_{1} C_{1}+l_{2} C_{12}\right)^{2}+\left(l_{1} S_{1}+l_{2} S_{12}\right)^{2}  \tag{2.7a}\\
& =l_{1}^{2}+l_{2}^{2}+2 l_{1} C_{1} l_{2} C_{12}+2 l_{1} S_{1} l_{2} S_{12} \tag{2.7b}
\end{align*}
$$

$\left[\because C_{1}^{2}+S_{1}^{2}=\cos ^{2} \theta_{1}+\sin ^{2} \theta_{2}=1\right.$ and $\left.C_{12}^{2}+S_{12}^{2}=\cos ^{2}\left(\theta_{1}+\theta_{2}\right)+\sin ^{2}\left(\theta_{1}+\theta_{2}\right)=1\right]$
(2.7c)

$$
\begin{align*}
X^{2}+ & Y^{2}=l_{1}^{2}+l_{2}^{2}+2 l_{1} l_{2}\left(C_{1} C_{12}+S_{1} S_{12}\right)  \tag{2.8a}\\
= & l_{1}^{2}+l_{2}^{2}+2 l_{1} l_{2}\left[\cos \theta_{1} \cos \left(\theta_{1}+\theta_{2}\right)+\sin \theta_{1} \sin \left(\theta_{1}+\theta_{2}\right)\right]  \tag{2.8b}\\
& =l_{1}^{2}+l_{2}^{2}+2 l_{1} l_{2} \cos \left(\theta_{1}-\theta_{2}-\theta_{1}\right)  \tag{2.8c}\\
& =l_{1}^{2}+l_{2}^{2}+2 l_{1} l_{2} \cos \theta_{2} \tag{2.8~d}
\end{align*}
$$

Hence using (2.8d)

$$
\begin{equation*}
\cos \theta_{2}=\frac{X^{2}+Y^{2}-l_{1}^{2}-l_{2}^{2}}{2 l_{1} l_{2}} \tag{2.9a}
\end{equation*}
$$

$$
\begin{equation*}
\theta_{2}=\cos ^{-1} \frac{X^{2}+Y^{2}-l_{1}^{2}-l_{2}^{2}}{2 l_{1} l_{2}} \tag{2.9b}
\end{equation*}
$$

Hence using (2.1) and (2.9b) the value of $\theta_{1}$ can be extracted as

$$
\begin{equation*}
\theta_{1}=\varphi-\theta_{2} \tag{2.10}
\end{equation*}
$$

The angular movement of each link length can be found out by using (2.9b), (2.10) which are inverse kinematics equations.

## 3. RESULTS AND DISCUSSION:

## Set of Data extracted from the forward kinematics mathematical modelling

Now the data collecting from equation (2.6a) and (2.6b) for forward kinematics have been tabulated in Table1. In the Table 1, the given data are L1, L2, $\theta_{1}$ and $\theta_{2}$, the corresponding extracted value of X and Y have been tabulated with help of forward kinematic modelling established in (2.6a) and (2.6b).
Table:1 Set of data extracted from forward kinematics mathematical modelling

| $\begin{gathered} \hline \mathrm{Sl} \\ \text { No } \end{gathered}$ | $\begin{aligned} & \hline \text { L1 } \\ & (\mathrm{cm}) \\ & \hline \end{aligned}$ | $\begin{aligned} & \hline \text { L2 } \\ & (\mathrm{cm}) \\ & \hline \end{aligned}$ | Ө1 (in degree) | $\theta 2$ (in degree) | $\begin{gathered} \hline \mathrm{X} \\ (\mathrm{~cm}) \\ \hline \end{gathered}$ | $\begin{gathered} \hline \mathrm{Y} \\ (\mathrm{~cm}) \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 8 | 5 | 20 | 22 | 11.23 | 6.08 |
| 2 | 8 | 5 | 25 | 10 | 11.35 | 6.23 |
| 3 | 8 | 5 | 28 | 32 | 9.56 | 8.08 |
| 4 | 8 | 5 | 30 | 20 | 10.14 | 7.83 |
| 5 | 8 | 5 | 35 | 38 | 8.01 | 9.37 |
| 6 | 8 | 5 | 45 | 30 | 6.95 | 10.49 |
| 7 | 8 | 5 | 47 | 50 | 4.84 | 10.81 |
| 8 | 8 | 5 | 52 | 43 | 4.49 | 11.28 |
| 9 | 8 | 5 | 60 | 65 | 1.13 | 11.02 |
| 10 | 8 | 5 | 63 | 72 | 0.096 | 10.66 |
| 11 | 8 | 5 | 74 | 50 | -0.59 | 11.83 |
| 12 | 8 | 5 | 80 | 60 | -2.44 | 11.09 |
| 13 | 8 | 5 | 83 | 37 | -1.52 | 12.27 |
| 14 | 8 | 5 | 85 | 90 | -4.3 | 8.4 |
| 15 | 8 | 5 | 95 | 100 | -5.53 | 3.29 |
| 16 | 10 | 8 | 20 | 22 | 15.34208481 | 8.773246284 |
| 17 | 10 | 8 | 25 | 10 | 15.61629422 | 8.814794108 |
| 18 | 10 | 8 | 28 | 32 | 12.82947593 | 11.62291886 |
| 19 | 10 | 8 | 30 | 20 | 13.80255492 | 11.12835554 |
| 20 | 10 | 8 | 35 | 38 | 10.53049408 | 13.38620241 |
| 21 | 10 | 8 | 45 | 30 | 9.141620173 | 14.79847442 |
| 22 | 10 | 8 | 47 | 50 | 5.845028853 | 15.25390623 |
| 23 | 10 | 8 | 52 | 43 | 5.459368811 | 15.84966512 |
| 24 | 10 | 8 | 60 | 65 | 0.411388509 | 15.21347039 |
| 25 | 10 | 8 | 63 | 72 | -1.11694925 | 14.56691949 |
| 26 | 10 | 8 | 74 | 50 | -1.71716967 | 16.24491754 |
| 27 | 12 | 7 | 20 | 22 | 16.47832523 | 8.788155964 |
| 28 | 12 | 7 | 25 | 10 | 16.60975775 | 9.086454195 |
| 29 | 12 | 7 | 28 | 32 | 14.09537111 | 11.69583658 |
| 30 | 12 | 7 | 30 | 20 | 14.89181811 | 11.3623111 |
| 31 | 12 | 7 | 35 | 38 | 11.87642646 | 13.57705053 |
| 32 | 12 | 7 | 45 | 30 | 10.29701469 | 15.24676216 |
| 33 | 12 | 7 | 47 | 50 | 7.330894917 | 15.72406748 |
| 34 | 12 | 7 | 52 | 43 | 6.777847505 | 16.42949193 |

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| 35 | 12 | 7 | 60 | 65 | 1.984964946 | 16.12636916 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 36 | 12 | 7 | 63 | 72 | 0.498138529 | 15.64182576 |
| 37 | 12 | 7 | 74 | 50 | -0.60670205 | 17.33840336 |
| 38 | 15 | 10 | 20 | 22 | 21.52683757 | 11.82160821 |
| 39 | 15 | 10 | 25 | 10 | 21.78613725 | 12.07503829 |
| 40 | 15 | 10 | 28 | 32 | 18.24421389 | 15.70232748 |
| 41 | 15 | 10 | 30 | 20 | 19.41825715 | 15.16044443 |
| 42 | 15 | 10 | 35 | 38 | 15.21099771 | 18.1666941 |
| 43 | 15 | 10 | 45 | 30 | 13.19479217 | 20.26585998 |
| 44 | 15 | 10 | 47 | 50 | 9.011281967 | 20.89576704 |
| 45 | 15 | 10 | 52 | 65 | 8.363364702 | 21.78210829 |
| 46 | 15 | 10 | 60 | 63 | 1.764235636 | 21.1819015 |
| 47 | 15 | 10 | 74 | 50 | -0.26121031 | 20.43616567 |
| 48 | 15 | 10 | 80 | 32 | -1.45736869 | 22.70930116 |
| 49 | 6 | 4 | 73 | 20 | -0.16858815 | 9.852757169 |
| 50 | 6 | 4 |  |  |  |  |

## Set of Data extracted from the inverse kinematics mathematical modelling

Now the data collecting from equation (2.9b) and (2.10) for inverse kinematics have been tabulated in Table2. In the Table 2, the given data are $\mathrm{L} 1, \mathrm{~L} 2, \mathrm{X}$ and Y , the corresponding extracted value of $\theta_{1}$ and $\theta_{2}$ have been tabulated with help of inverse kinematic modelling established in (2.9b) and (2.10).
Table:2 Set of data extracted from inverse kinematics mathematical modelling

| $\begin{gathered} \mathrm{Sl} \\ \mathrm{No} . \end{gathered}$ | $\begin{gathered} \mathrm{X} \\ (\mathrm{~cm}) \\ \hline \end{gathered}$ | $\begin{gathered} \mathrm{Y} \\ (\mathrm{~cm}) \end{gathered}$ | $\begin{gathered} \text { L1 } \\ (\mathrm{cm}) \\ \hline \end{gathered}$ | $\begin{gathered} \mathrm{L} 2 \\ (\mathrm{~cm}) \\ \hline \end{gathered}$ | $\begin{gathered} \hline \Phi \\ \text { (in degree) } \end{gathered}$ | $\overline{\Theta 1}$ <br> (in degree) | $\overline{\theta 2}$ <br> (in degree) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 11.23 | 6.08 | 8 | 5 | 42 | 20 | 22 |
| 2 | 11.35 | 6.23 | 8 | 5 | 35 | 25 | 10 |
| 3 | 9.56 | 8.08 | 8 | 5 | 60 | 28 | 32 |
| 4 | 10.14 | 7.83 | 8 | 5 | 50 | 30 | 20 |
| 5 | 8.01 | 9.37 | 8 | 5 | 73 | 35 | 38 |
| 6 | 6.95 | 10.49 | 8 | 5 | 75 | 45 | 30 |
| 7 | 4.84 | 10.81 | 8 | 5 | 97 | 47 | 50 |
| 8 | 4.49 | 11.28 | 8 | 5 | 95 | 52 | 43 |
| 9 | 1.13 | 11.02 | 8 | 5 | 125 | 60 | 65 |
| 10 | 0.096 | 10.66 | 8 | 5 | 135 | 63 | 72 |
| 11 | -0.59 | 11.83 | 8 | 5 | 124 | 74 | 50 |
| 12 | -2.44 | 11.09 | 8 | 5 | 140 | 80 | 60 |
| 13 | -1.52 | 12.27 | 8 | 5 | 120 | 83 | 37 |
| 14 | -4.3 | 8.4 | 8 | 5 | 175 | 85 | 90 |
| 15 | -5.53 | 3.29 | 8 | 5 | 195 | 95 | 100 |
| 16 | 15.34208481 | 8.773246284 | 10 | 8 | 42 | 20 | 22 |
| 17 | 15.61629422 | 8.814794108 | 10 | 8 | 35 | 25 | 10 |
| 18 | 12.82947593 | 11.62291886 | 10 | 8 | 60 | 28 | 32 |
| 19 | 13.80255492 | 11.12835554 | 10 | 8 | 50 | 30 | 20 |
| 20 | 10.53049408 | 13.38620241 | 10 | 8 | 73 | 35 | 38 |
| 21 | 9.141620173 | 14.79847442 | 10 | 8 | 75 | 45 | 30 |


| 22 | 5.845028853 | 15.25390623 | 10 | 8 | 97 | 47 | 50 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 23 | 5.459368811 | 15.84966512 | 10 | 8 | 95 | 52 | 43 |
| 24 | 0.411388509 | 15.21347039 | 10 | 8 | 125 | 60 | 65 |
| 25 | -1.11694925 | 14.56691949 | 10 | 8 | 135 | 63 | 72 |
| 26 | -1.71716967 | 16.24491754 | 10 | 8 | 124 | 74 | 50 |
| 27 | 16.47832523 | 8.788155964 | 12 | 7 | 42 | 20 | 22 |
| 28 | 16.60975775 | 9.086454195 | 12 | 7 | 35 | 25 | 10 |
| 29 | 14.09537111 | 11.69583658 | 12 | 7 | 60 | 28 | 32 |
| 30 | 14.89181811 | 11.3623111 | 12 | 7 | 50 | 30 | 20 |
| 31 | 11.87642646 | 13.57705053 | 12 | 7 | 73 | 35 | 38 |
| 32 | 10.29701469 | 15.24676216 | 12 | 7 | 75 | 45 | 30 |
| 33 | 7.330894917 | 15.72406748 | 12 | 7 | 97 | 47 | 50 |
| 34 | 6.777847505 | 16.42949193 | 12 | 7 | 95 | 52 | 43 |
| 35 | 1.984964946 | 16.12636916 | 12 | 7 | 125 | 60 | 65 |
| 36 | 0.498138529 | 15.64182576 | 12 | 7 | 135 | 63 | 72 |
| 37 | -0.60670205 | 17.33840336 | 12 | 7 | 124 | 74 | 50 |
| 38 | 21.52683757 | 11.82160821 | 15 | 10 | 42 | 20 | 22 |
| 39 | 21.78613725 | 12.07503829 | 15 | 10 | 35 | 25 | 10 |
| 40 | 18.24421389 | 15.70232748 | 15 | 10 | 60 | 28 | 32 |
| 41 | 19.41825715 | 15.16044443 | 15 | 10 | 50 | 30 | 20 |
| 42 | 15.21099771 | 18.1666941 | 15 | 10 | 73 | 35 | 38 |
| 43 | 13.19479217 | 20.26585998 | 15 | 10 | 75 | 45 | 30 |
| 44 | 9.011281967 | 20.89576704 | 15 | 10 | 97 | 47 | 50 |
| 45 | 8.363364702 | 21.78210829 | 15 | 10 | 95 | 52 | 43 |
| 46 | 1.764235636 | 21.1819015 | 15 | 10 | 125 | 60 | 65 |
| 47 | -0.26121031 | 20.43616567 | 15 | 10 | 135 | 63 | 72 |
| 48 | -1.45736869 | 22.70930116 | 15 | 10 | 124 | 74 | 50 |
| 49 | 0.456537308 | 9.61758 | 6 | 4 | 112 | 80 | 32 |
| 50 | -0.1685881 | 9.852757 | 6 | 4 | 103 | 83 | 20 |

Theses set of values in Tables 1 and 2 can be used in future for the prediction purpose by ANN methodology [12].

## 4. CONCLUSION:

The present study gives the idea of the mathematical expression of the inverse and forward kinematics modelling of the 2-R serial manipulator. The corresponding mathematical modelling gives the extracted output values and have been tabulated. These set of data can be implemented for the prediction purpose of the kinematics modelling of this manipulator. These data can be further used for the testing and validation of ANN. The prediction can help before the real time works.

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