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Research Article

Inverse and Forward Kinematics of 2R Planner Serial Robot

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Abstract: The robots are essential in the modern industries and manufacturing field for precise activities. Robots are replacing human work force to a huge extent. The study of robotics is now an emerging filed in the research area. The robot has two types of as serial and parallel manipulator. In this work the serial manipulator of 2DOF planner robot has been considered. In the present paper, the kinematics modelling of a 2R planner robot has been done for the inverse and forward kinematics. The detailed mathematical modelling of these kinematics equations can help to derive the similar type of mathematical modelling for higher degree of freedom robot. The different set of output value of corresponding kinematics modelling have been tabulated in this work.

Key Words: 2-R Planner Robot, Serial Robot, Inverse kinematics, Forward kinematics

1. INTRODUCTION:

Now a days robots are seen to be replacing the human interactions in almost every sphere of life. In almost every kind of industrial applications like production, packaging, logistics etc. Robots are replacing human work force to a huge extent. Hence the study of robot and their kinematics have emerged out to be a very important subject in the field of mechanical engineering and mathematical analysis. In the study of robotics, application of mechanical, electrical, electronics, computer science engineering is seen almost equivalently [1-2]. The mathematical modelling of inverse kinematics of the robotics is required to find out the end effector position with help of each link length and movement [2-4]. The mathematical modelling of forward kinematics of the robotics is essential to extract each link movement with help of the end effector position and each link length [1, 4-5]. But apart from this studies a huge ocean of mathematical applications are also seen in this field such as DH Matrix analysis for the inverse and forward kinematic equations of robot or the complicated algebraic analysis for the analysis of the same [5-6]. These type of kinematics equations can be helpful for the idea of the parallel manipulators [7-9]. The control of the motion of these manipulator can be done with help of modern controllers [10-11]. The mathematical modelling can be implemented to get the prior knowledge before real-time experimental works. In this research work, the basic of the 2-R planner serial robot has been discussed in the next section. The mathematical modelling of forward and inverse kinematics for 2R serial manipulator have been formulated step by step in the next section 3. The different set of values extracted from the inverse and forward kinematics have been tabulated in the section 4. Theses set of values can be used in future for the prediction purpose by ANN methodology [12].

2. 2R PLANNER ROBOT:

2R Planner manipulator is called 2R because it has 2 revolving links. It has only rotary motion and no linear motions. It is called planner because its motion occurs only in 1 plane. This is a typical manipulator which is the precursors of all types of manipulators.



Figure 1. 2R serial planner manipulator.

The Fig. 1 depicts the 2R planner manipulator with link length l_1 , and l_2 of link 1 and 2 respectively and θ_1 and θ_2 are angle of l_1 w.r.t X-axis and l_2 w.r.t l_1 respectively. The end effector position have been taken here as X, Y and φ

where
$$\varphi = \theta_1 + \theta_2$$

(2.1)

X and Y are the coordinate in x and y axis respectively.

2.1. Mathematical Modelling of Forward and Inverse Kinematics of 2R planner robot Forward Kinematics

In forward kinematic analysis the angular positions of the links θ_1 and θ_2 are given and the position of the end effector (X,Y) have to be found out. The X & Y-coordinate of the end effector is basically the summation of the X & Y-coordinates of the individual links, hence

$$X = X_{l_1} + X_{l_2} \tag{2.2}$$

Now from the Fig.1 it can be written that

$$X_{l_1} = l_1 \cos\theta_1 = l_1 C_1 \tag{2.3a}$$

$$X_{l_2} = l_2 \cos(\theta_1 + \theta_2) = l_2 C_{12}$$
(2.3b)

Similarly,

$$Y = Y_{l_1} + Y_{l_2} \tag{2.4}$$

$$Y_{l_1} = l_1 \sin \theta_1 = l_1 S_1$$
 (2.5a)

$$Y_{l_2} = l_2 \sin(\theta_1 + \theta_2) = l_2 S_{12}$$
(2.5a)

So it can be written from (2.2, 2.3a-b) and (2.4, 2.5a-b)

$$X = l_1 C_1 + l_2 C_{12} (2.6a)$$

$$Y = l_1 S_1 + l_2 S_{12} \tag{2.6b}$$

The end effector position can be extracted by using (2.6a) and (2.6b) which are forward kinematics equations.

Inverse Kinematics:

Hence using (2.8d)

In inverse kinematics the position of the end effector (X, Y) are given and θ_1 and θ_2 have to be found out. From (2.6a) and (2.6b) it can be written that

$$X^{2} + Y^{2} = (l_{1}C_{1} + l_{2}C_{12})^{2} + (l_{1}S_{1} + l_{2}S_{12})^{2}$$
(2.7a)

$$= l_1^2 + l_2^2 + 2l_1C_1l_2C_{12} + 2l_1S_1l_2S_{12}$$
(2.7b)

[:: $C_1^2 + S_1^2 = \cos^2\theta_1 + \sin^2\theta_2 = 1$ and $C_{12}^2 + S_{12}^2 = \cos^2(\theta_1 + \theta_2) + \sin^2(\theta_1 + \theta_2) = 1$] (2.7c)

$$X^{2} + Y^{2} = l_{1}^{2} + l_{2}^{2} + 2l_{1}l_{2}(C_{1}C_{12} + S_{1}S_{12})$$
(2.8a)

$$= l_1^2 + l_2^2 + 2l_1l_2[\cos\theta_1\cos(\theta_1 + \theta_2) + \sin\theta_1\sin(\theta_1 + \theta_2)]$$
(2.8b)

$$= l_1^2 + l_2^2 + 2l_1 l_2 \cos(\theta_1 - \theta_2 - \theta_1)$$
(2.8c)

$$= l_1^2 + l_2^2 + 2l_1 l_2 \cos\theta_2 \tag{2.8d}$$

$$\cos\theta_2 = \frac{X^2 + Y^2 - l_1^2 - l_2^2}{2l_1 l_2} \tag{2.9a}$$

$$\theta_2 = \cos^{-1} \frac{X^2 + Y^2 - l_1^2 - l_2^2}{2l_1 l_2} \tag{2.9b}$$

Hence using (2.1) and (2.9b) the value of θ_1 can be extracted as $\theta_1 = \varphi - \theta_2$

(2.10)





The angular movement of each link length can be found out by using (2.9b), (2.10) which are inverse kinematics equations.

3. RESULTS AND DISCUSSION:

Set of Data extracted from the forward kinematics mathematical modelling

Now the data collecting from equation (2.6a) and (2.6b) for forward kinematics have been tabulated in Table1. In the Table 1, the given data are L1, L2, θ_1 and θ_2 , the corresponding extracted value of X and Y have been tabulated with help of forward kinematic modelling established in (2.6a) and (2.6b).

Sl No	L1	L2	θ1	θ2	X	Y
	(cm)	(cm)	(in degree)	(in degree)	(cm)	(cm)
1	8	5	20	22	11.23	6.08
2	8	5	25	10	11.35	6.23
3	8	5	28	32	9.56	8.08
4	8	5	30	20	10.14	7.83
5	8	5	35	38	8.01	9.37
6	8	5	45	30	6.95	10.49
7	8	5	47	50	4.84	10.81
8	8	5	52	43	4.49	11.28
9	8	5	60	65	1.13	11.02
10	8	5	63	72	0.096	10.66
11	8	5	74	50	-0.59	11.83
12	8	5	80	60	-2.44	11.09
13	8	5	83	37	-1.52	12.27
14	8	5	85	90	-4.3	8.4
15	8	5	95	100	-5.53	3.29
16	10	8	20	22	15.34208481	8.773246284
17	10	8	25	10	15.61629422	8.814794108
18	10	8	28	32	12.82947593	11.62291886
19	10	8	30	20	13.80255492	11.12835554
20	10	8	35	38	10.53049408	13.38620241
21	10	8	45	30	9.141620173	14.79847442
22	10	8	47	50	5.845028853	15.25390623
23	10	8	52	43	5.459368811	15.84966512
24	10	8	60	65	0.411388509	15.21347039
25	10	8	63	72	-1.11694925	14.56691949
26	10	8	74	50	-1.71716967	16.24491754
27	12	7	20	22	16.47832523	8.788155964
28	12	7	25	10	16.60975775	9.086454195
29	12	7	28	32	14.09537111	11.69583658
30	12	7	30	20	14.89181811	11.3623111
31	12	7	35	38	11.87642646	13.57705053
32	12	7	45	30	10.29701469	15.24676216
33	12	7	47	50	7.330894917	15.72406748
34	12	7	52	43	6.777847505	16.42949193

 Table:1 Set of data extracted from forward kinematics mathematical modelling



35	12	7	60	65	1.984964946	16.12636916
36	12	7	63	72	0.498138529	15.64182576
37	12	7	74	50	-0.60670205	17.33840336
38	15	10	20	22	21.52683757	11.82160821
39	15	10	25	10	21.78613725	12.07503829
40	15	10	28	32	18.24421389	15.70232748
41	15	10	30	20	19.41825715	15.16044443
42	15	10	35	38	15.21099771	18.1666941
43	15	10	45	30	13.19479217	20.26585998
44	15	10	47	50	9.011281967	20.89576704
45	15	10	52	43	8.363364702	21.78210829
46	15	10	60	65	1.764235636	21.1819015
47	15	10	63	72	-0.26121031	20.43616567
48	15	10	74	50	-1.45736869	22.70930116
49	6	4	80	32	-0.45653730	9.617581936
50	6	4	83	20	-0.16858815	9.852757169

Set of Data extracted from the inverse kinematics mathematical modelling

Now the data collecting from equation (2.9b) and (2.10) for inverse kinematics have been tabulated in Table2. In the Table 2, the given data are L1, L2, X and Y, the corresponding extracted value of θ_1 and θ_2 have been tabulated with help of inverse kinematic modelling established in (2.9b) and (2.10).

Table:2 Set of data extracted from inverse kinematics mathematical modelling

SI	Х	Y	L1	L2	Φ	θ1	θ2
NO.	(am)	(am)	(am)	(am)	(in degree)	(in dagraa)	(in dagraa)
1	11.23	6.08	8	5	42	20	22
2	11.35	6.23	8	5	35	25	10
3	9.56	8.08	8	5	60	28	32
4	10.14	7.83	8	5	50	30	20
5	8.01	9.37	8	5	73	35	38
6	6.95	10.49	8	5	75	45	30
7	4.84	10.81	8	5	97	47	50
8	4.49	11.28	8	5	95	52	43
9	1.13	11.02	8	5	125	60	65
10	0.096	10.66	8	5	135	63	72
11	-0.59	11.83	8	5	124	74	50
12	-2.44	11.09	8	5	140	80	60
13	-1.52	12.27	8	5	120	83	37
14	-4.3	8.4	8	5	175	85	90
15	-5.53	3.29	8	5	195	95	100
16	15.34208481	8.773246284	10	8	42	20	22
17	15.61629422	8.814794108	10	8	35	25	10
18	12.82947593	11.62291886	10	8	60	28	32
19	13.80255492	11.12835554	10	8	50	30	20
20	10.53049408	13.38620241	10	8	73	35	38
21	9.141620173	14.79847442	10	8	75	45	30



22	5.845028853	15.25390623	10	8	97	47	50
23	5.459368811	15.84966512	10	8	95	52	43
24	0.411388509	15.21347039	10	8	125	60	65
25	-1.11694925	14.56691949	10	8	135	63	72
26	-1.71716967	16.24491754	10	8	124	74	50
27	16.47832523	8.788155964	12	7	42	20	22
28	16.60975775	9.086454195	12	7	35	25	10
29	14.09537111	11.69583658	12	7	60	28	32
30	14.89181811	11.3623111	12	7	50	30	20
31	11.87642646	13.57705053	12	7	73	35	38
32	10.29701469	15.24676216	12	7	75	45	30
33	7.330894917	15.72406748	12	7	97	47	50
34	6.777847505	16.42949193	12	7	95	52	43
35	1.984964946	16.12636916	12	7	125	60	65
36	0.498138529	15.64182576	12	7	135	63	72
37	-0.60670205	17.33840336	12	7	124	74	50
38	21.52683757	11.82160821	15	10	42	20	22
39	21.78613725	12.07503829	15	10	35	25	10
40	18.24421389	15.70232748	15	10	60	28	32
41	19.41825715	15.16044443	15	10	50	30	20
42	15.21099771	18.1666941	15	10	73	35	38
43	13.19479217	20.26585998	15	10	75	45	30
44	9.011281967	20.89576704	15	10	97	47	50
45	8.363364702	21.78210829	15	10	95	52	43
46	1.764235636	21.1819015	15	10	125	60	65
47	-0.26121031	20.43616567	15	10	135	63	72
48	-1.45736869	22.70930116	15	10	124	74	50
49	0.456537308	9.61758	6	4	112	80	32
50	-0.1685881	9.852757	6	4	103	83	20

Theses set of values in Tables 1 and 2 can be used in future for the prediction purpose by ANN methodology [12].

4. CONCLUSION:

The present study gives the idea of the mathematical expression of the inverse and forward kinematics modelling of the 2-R serial manipulator. The corresponding mathematical modelling gives the extracted output values and have been tabulated. These set of data can be implemented for the prediction purpose of the kinematics modelling of this manipulator. These data can be further used for the testing and validation of ANN. The prediction can help before the real time works.

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