INTERNATIONAL JOURNAL FOR INNOVATIVE RESEARCH IN MULTIDISCIPLINARY FIELD
ISSN: 2455-0620 [Impact Factor: 6.719]
Monthly, Peer-Reviewed, Refereed, Indexed Journal with IC Value : 86.87
Volume - 8, Issue - 5, MAY - 2022 Publication Date: 31/05/2022

DOIs:10.2015/IJIRMF/202205032 --:-- Research Article

Existence of Massless Scalar Field Coupled with Gravitational and Electromagnetic Field

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Abstract: *We established the existence of plane scalar waves coupled with gravitational and electromagnetic waves. Further the existence of scalar waves with gravitational and electromagnetic waves are related to the eigen values of energy momentum tensor which are all zero.*

Key Words: *Gravitational wave, Scalar wave, electromagnetic wave.*

1. INTRODUCTION :

The nature of plane gravitational waves co-existing with electromagnetic waves in four dimensional space-time V_4 is mathematically investigated by Takeno [1]. According to Kaluza-Klein, the idea that the world has more than four dimensions with luminous insight to realize that the five dimensional manifold can be used to unite Einstein theory of general relativity with the Maxwell's theory of electromagnetic. During the last few years much of the work is aimed at extending our knowledge of the gravitational and other fields to develop theories in more than four dimensions in general relativity. Therefore, Thengane et al. [2] reformulated Takeno's definition of plane waves in V_4 to five dimensional space-time having two time axes. Ambatkar et al. [3] elaborated the work of Thengane's for $Z = z - \frac{1}{\sqrt{2}}(t_1 - t_2)$ type waves and obtained the results for non-vanishing components of Ricci tensor R_{ij}, energy momentum tensor T_{ij}, and electromagnetic energy tensor E_{ij} as,

$$R_{33} = -\sqrt{2}\,R_{34} = \sqrt{2}\,R_{35} = 2\,R_{44} = -2\,R_{45} = 2\,R_{55}$$
$$T_{33} = -\sqrt{2}\,T_{34} = \sqrt{2}\,T_{35} = 2\,T_{44} = -2\,T_{45} = 2\,T_{55}$$
$$E_{33} = -\sqrt{2}\,E_{34} = \sqrt{2}\,E_{35} = 2\,E_{44} = -2\,E_{45} = 2\,E_{55}$$

In section 2, we add the massless scalar field with the gravitational and electromagnetic field and studied its consequences. It has been established that, the scalar waves and electromagnetic waves with the gravitational waves exist. Further the existence of scalar waves with the gravitational and electromagnetic waves are related to the zero eigen values of energy momentum tensor T_{ij}. The concluding remarks are given in last **section 3**. This work included in [5]
For our study, we employ the field equations of general relativity

$$R_{ij} = -8\pi\left(T_{ij} - \frac{1}{2}\,g_{ij}\,T\right), \qquad i,j = 1,2\,..5 \tag{1.1}$$

and the line element

$$\begin{aligned} ds^2 = &-Adx^2 - 2Ddxdy - Bdy^2 + 2(-C+E)dz^2 - 2\sqrt{2}\,Edzdt_1 \\ &+ (C+E)\,dt_1^2 + 2\sqrt{2}\,(-C+F)dz\,dt_2 - 2\,F\,dt_1dt_2 \\ &+ (E+F)\,dt_2^2 \end{aligned} \tag{1.2}$$

Where A, B, C, D, E, F are function of $Z = z - \frac{1}{\sqrt{2}}(t_1 - t_2)$

2. MASSLESS SCALAR FIELD AND ELECTROMAGNETIC FIELD COUPLED WITH GRAVITATIONAL FIELD

The matter tensor is described by the tensor

$$T_{ij} = E_{ij} + \frac{1}{4\pi}\left[V_i\,V_j - \frac{1}{2}\,g_{ij}\,V_s\,V^s\right]$$

Where, $E_{ij} = \frac{1}{4\pi}\left[\frac{1}{4}\, g_{ij}\, F_{sp}\, F^{sp} - F_{is}\, F_j^s\right],\ p = 1, \ldots 5$ (2.1)

Here F_{is} is an electromagnetic Maxwell skew tensor given by

$$F_{is} = K_{s;i} - K_{i;s}$$
$$= K_{s,i} - K_{i,s}\,, \quad K_i = K_i(Z)$$

Semi colon (;) means covariant derivative with respect to g_{ij} and the comma (,) stands for partial derivative with respect to x^i.

The gravitational field is given by (1.2).

As, $V_s\, V^s = 0$, $T_{ij} = E_{ij} + \frac{1}{4\pi}\,(V_i\, V_j)$

and $R_{ij} = -8\pi\left[E_{ij} + \frac{1}{4\pi}\left(V_i\, V_j\right) - \frac{1}{2}\, g_{ij}\, T\right]$, due to (1.1) (2.2)

Now, $T = T_i^i = E_i^i + \frac{1}{4\pi}\, V_i\, V^I = 0$

Then $R_{ij} = -8\pi\left[E_{ij} + \frac{1}{4\pi}\,(V_i\, V_j)\right]$, using (2.2) and T

$$= (-2)\left[4\pi\, E_{ij} + (\, V_i\, V_j\,)\right] \quad (2.3)$$

We have

$$E_{33} = -\sqrt{2}\, E_{34} = \sqrt{2}\, E_{35} = 2\, E_{44} = -2\, E_{45} = 2\, E_{55}\,, \quad \text{Ambatkar [3]} \quad (2.4)$$

Equation (2.3) and (2.4) gives,

$$R_{33} = -\sqrt{2}\, R_{34} = \sqrt{2}\, R_{35} = 2\, R_{44} = -2\, R_{45} = 2\, R_{55} \quad (2.5)$$

For the line element (1.2), we have

$$R_{33} = -\sqrt{2}\, R_{34} = \sqrt{2}\, R_{35} = 2\, R_{44} = -2\, R_{45} = 2\, R_{55}\,, \quad \text{Ambatkar [3]} \quad (2.6)$$

Equations (2.5) are compatible with equation (2,6). Hence it follows the existence of scalar waves coupled with gravitational and electromagnetic waves.

The eigen values are given by

$$\left|T_{ij} - \lambda\, g_{ij}\right| = 0$$
$$\Rightarrow \lambda^2\,(AB - D^2)(128\,\pi^3\,\lambda^3\,C^2)(2E + C - F) = 0$$
$$\Rightarrow \lambda^5 = 0\,, \text{ as } -D^2 \neq 0,\ C^2\,(2E + C - F)\ \neq 0\,, \quad \text{Ambatkar [3]}$$

Implying that all eigen values are equal (= 0). Thus, the coexistence of scalar waves with gravitational and electromagnetic waves is related to the eigen values of T_{ij}, which are all zero. The inference drawn from all above cases can be reformulated into the following conclusion.

3. CONCLUSION :

The existence of massless scalar waves coupled with gravitational and electromagnetic waves is guaranteed by five zero eigen values of the energy momentum tensor of the combined distribution of scalar and electromagnetic fields for $Z = z - \frac{1}{\sqrt{2}}\,(t_1 - t_2)$ type waves.

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