



COMPARING THE SHARPENESS OF SHARPE'S AND SORTINO'S RATIOS IN PORTFOLIO CONSTRUCTION

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Abstract: *The process of portfolio construction remains an evergreen topic of research due to the availability of numerous performance measures and criteria for selecting the investment alternative. In the modern world of software revolution, framing a portfolio without using a mathematical framework has become an outdated idea. This study aims at comparing two different objectives in the process of constructing a portfolio: Maximizing the Sharpe's ratio and maximizing the Sortino's ratio. Sharpe's ratio which is a reward to volatility ratio is widely used as a performance measure in the process of portfolio construction. In this ratio, the volatility is nothing but the standard deviation of the returns. The serious drawback of standard deviation is that it considers both negative and positive deviations. But while measuring the risk, it would be more appropriate if negative deviations alone are considered. This is achieved through Sortino's ratio and the study attempts to test the credibility of Sortino's ratio over Sharpe's ratio using a Linear Programming Model. The study concludes that the usage of Sortino's ratio leads to maximization of returns and minimization of risk.*

Key Words: *Portfolio, Sharpe's ratio, Sortino's ratio, Excess Returns, Negative excess returns, volatility, Linear Programming.*

1. INTRODUCTION:

From the perception of an investor, investment allows the money to grow. It is also true that the process of investment oils the economic wheels of a country. Investing in a single alternative is not favored by investors because they like to diversify the risk. Hence framing a portfolio has become inevitable. The process of portfolio construction is complicated because of the difficulty in

- Choosing the right alternatives
- Choosing the right performance measures
- Choosing the right mathematical model
- Choosing the right number of alternatives
- Choosing the weights for each alternative.

There are numerous forms of mathematical models, alternatives and performance measures due to which the process of portfolio construction demands greater expertise. Mathematical models enjoy a greater role in this process because of their capacity to consider all possible alternatives and also to identify the best alternative scientifically. This study tries to explore the form of models widely used in practice and with this focus, the following literature is reviewed. The process of Portfolio construction assumed scientific form with the development of the conceptual framework by Markowitz in 1952. The process which was done without any scientific basis became an attractive area of research after this contribution. This framework gives a complete summary of the problem statement along with a scientifically derived solution to the statement.

In 1963, another hallmark took place with the introduction of Single Index model by Sharpe. This study tried to understand the movement of stock prices based on the movement of the market index. The attitude of players of the stock market generally leads to overpricing or under pricing of stocks. This when identified may help the investor to make right investment decisions. The Capital Asset Pricing Model (CAPM) developed by Sharpe et al in 1964 established the relationship between the return and risk and it gives a framework to identify whether the assets are under priced or overpriced.



Treynor (1965) developed a ratio which cancels the unsystematic risk due to diversification. Jensen in 1968 has developed a measure based on differential returns. It is the difference between the actual returns and the expected returns of a portfolio calculated by considering the risk of the portfolio. The Sharpe's ratio developed by Sharpe in 1966 also called as a reward to risk ratio helped in choosing the securities in a relative manner. This development was seen as a major breakthrough and even today this ratio is widely used in ranking the securities. The works which deserve special mentioning are by Fama and Macbeth, 1973 and Rosenberg, 1998 explores the various implications of CAPM using historical rates of return and market returns using cross sectional and time series regression analysis.

Frank Sortino (1994 and 1999) recognized the relevance of introducing investor's risk preferences into performance measures with the introduction of the downside risk concept into the performance measurement literature. Downside risk incorporates the risk preferences of the investor by introducing a minimal acceptable rate of return (MAR), which represents the objective of the investor. Good volatility (above the MAR) is distinguished from Bad volatility (below the MAR). Therefore, volatility per se, is not synonymous with risk.

Subathra.R (2022) has proved that while using a Linear Programming framework for constructing an optimal portfolio, the objective of a risk averting investor should be to minimize the risk and for a risk taking investor, the objective should be to maximize the returns.

2. RESEARCH METHODOLOGY:

A portfolio is simply a collection of assets, characterized by mean, variances and co variances of their returns. The mean return of i th asset is denoted as r_i . The variances and co variances of n assets are represented in the following matrix.

	r_1	r_2	r_3	---	r_n
r_1	σ_1^2	σ_{12}	σ_{13}	---	σ_{1n}
r_2	σ_{21}	σ_2^2	σ_{23}	---	σ_{2n}
r_3	σ_{31}	σ_{32}	σ_3^2	--	σ_{3n}
-	--	--	--	--	-
r_n	σ_{n1}	σ_{n2}	σ_{n3}	--	σ_n^2

Thus the covariance of an asset with itself is the variance. $\sigma_{nn} = \sigma_n^2$

If n assets each with weight w_1, w_2, \dots, w_n are considered such that $\sum_{i=1}^n w_i = 1$ the expected portfolio return is derived as

$$\bar{r}_p = w_1\bar{r}_1 + w_2\bar{r}_2 + \dots + w_n\bar{r}_n = \sum_{i=1}^n w_i\bar{r}_i \quad \text{-----(1)}$$

The variance of the portfolio return with two assets is

$$\sigma_p^2 = Var(\bar{r}_p) = \sum_{i=1}^n \sum_{j=1}^n w_i w_j \sigma_{ij} \quad \text{-----(2)}$$

$$\text{where } \sigma_{ii} = \sigma_i^2 \quad \text{-----(3)}$$

$$\text{The volatility of the portfolio return is } \sigma_p = \sqrt{Var(\bar{r}_p)} = \sqrt{\sigma_p^2} \quad \text{-----(4)}$$

In an equally weighted portfolio of n assets, the variance term is $(1/n)^2 \sigma_{ii}$ and the covariance term is $(1/n)^2 \sigma_{ij}$. Adding all the terms

$$\begin{aligned} \sigma_p^2 &= \sum_{i=1}^n \sum_{j=1}^n w_i w_j \sigma_{ij} = \sum_{i=1}^n \left(\frac{1}{n}\right)^2 \sigma_{ii} + \sum_{i=1}^n \sum_{j \neq i=1}^n \left(\frac{1}{n}\right)^2 \sigma_{ij} \\ &= \left(\frac{1}{n}\right) \left(\frac{1}{n} \sum_{i=1}^n \sigma_i^2\right) + \left(\frac{n^2-n}{n^2}\right) \left(\frac{1}{n^2-n} \sum_{i=1}^n \sum_{j \neq i=1}^n \sigma_{ij}\right) \\ &= \left(\frac{1}{n}\right) * \text{average variance} + \left(\frac{n^2-n}{n^2}\right) * \text{Average covariance} \quad \text{----- (5)} \end{aligned}$$

As n becomes very large, the contribution of variance terms goes to zero and the contribution of covariance terms goes to the average covariance. The risk and return characteristics of a security cannot be predicted in advance which makes investment decision a complicated problem. The investors manage this situation by holding a portfolio which diversifies the risk. But diversification is also a tedious task which requires decision making in terms of types of securities to be included in the portfolio and the number of securities to be included in the portfolio. Even after finding the types of securities and the number of securities, finding the proportion of money to be invested in each security remains a complicated problem. To facilitate the availability of these weights, it is necessary to have a mathematical framework which considers all possible alternatives and identifies the best alternative. But the real problem lies with the formulation of this mathematical framework. Subathra (2022) uses a Linear programming framework and suggests that the objective of a risk averting investor should be to minimize the risk. An extended



work by Subathra (2022) suggests that the objective of maximizing the Sharpe's ratio is more credible than the objective of minimizing the risk.

Adjusting the returns for risk gives a performance measure for a security. The Sharpe's ratio divides the reward which is the excess of risk-free rate by the standard deviation of returns. It is computed as *Sharpe's Ratio* = $\frac{r_p - r_f}{\sigma_p}$. Here r_p is the Portfolio return, r_f is the risk free return and σ_p is the portfolio standard deviation computed using the variance Co-variance matrix of excess returns.

Developed in 1966, this measure has proved to be effective in ranking the securities. But this measure has some drawbacks. It does not distinguish between upside and downside volatility. The denominator of the ratio gets increased in the presence of high outlier returns due to which the ratio gets reduced. This can be avoided by truncating the largest positive returns but the investors who like largest positive returns consider this truncation as illogical. Thus a security with positive skewness will be eliminated by the Sharpe's value but the inclusion of this security may increase the portfolio returns with minimum risk. Also if the return distribution is negatively skewed, the situation will be more risky than it is suggested by Sharpe's ratio. Thus if the returns have skewness, the usage of Sharpe's ratio may not be fruitful. Another reason which questions the usage of Sharpe's ratio is with respect to the way in which it calculates the risk. Sharpe's ratio considers the standard deviation of returns as risk. But risk for an investor means only decrease in returns. Hence the standard deviation which considers both positive and negative deviation for computing the risk is considered inappropriate. Even Markowitz (1959) recognizes the downside deviation as risk in his theory. Instead of standard deviation, a measure of dispersion which is computed by considering the negative deviations alone is taken as risk in Sortino's ratio. While doing so, the positive deviations are taken as 0. To this redefined deviated the formula for standard deviation is applied and it is called Downside risk (DR). Hence the Sortino's ratio is defined as

Sortino's Ratio = $\frac{r_p - r_f}{DR}$. Here r_p is the Portfolio return, r_f is the risk free return and DR is the portfolio standard deviation computed using the variance Co-variance matrix of negative excess returns. In order to compare the credibility of Sharpe's ratio and Sortino's ratio two models named as MaxSharpe and MaxSortino are framed in this study. In the first model the objective is to maximize the Sharpe's ratio. This ratio measures the additional return an investor earns by taking additional risk. It allows us to add new assets which can have a positive effect without adding any undue risk. In the second model the objective is to maximize the Sortino's ratio. The models which compute the proportion of amount to be invested in various securities are formed as LPP as follows.

LPP for maximizing the Sharpe's ratio (MaxSharpe Model)

$$\text{Maximise } SR = \frac{\bar{r}_p - r_f}{\sigma_p}$$

subject to $\sum_{i=1}^n w_i = 1$
 $w_i \geq 0$ for all i

LPP for maximizing the Sortino's ratio (MaxSortino Model)

$$\text{Maximise } SR = \frac{\bar{r}_p - r_f}{DR}$$

subject to $\sum_{i=1}^n w_i = 1$
 $w_i \geq 0$ for all i

3. RESULTS AND DISCUSSION:

The data used in this study is the secondary data collected from the official website of National Stock Exchange. Monthly closing prices of the randomly selected stocks from January 2016 to January 2022 are used to form the portfolios. Five portfolios are formed with randomly included stocks. Each portfolio consists of five stocks. The Linear programming problem formulated for each of the five portfolios is solved using Excel Solver which uses the method of simulation. The portfolios are constructed with randomly selected stocks in order to identify the credibility of the two different objectives: Maximise Sharpe's ratio and Maximise Sortino's ratio. The following table gives the summary values of the stocks included in the study.

Table-1: Summary Values- Close Prices of the securities

MEASURES	Mean	Standard Deviation	Kurtosis	Skewness
BAJAJFIN	3044.57	1950.4651	-0.0810	0.8829
INDUSIND	1238.78	403.7938	-0.6013	-0.2860
BHARTIARTEL	414.69	122.7765	-0.0902	0.9096



GRASIM	945.53	325.8976	0.3388	1.0280
INFY	778.62	409.5869	0.7377	1.3630
ONGC	120.87	27.4487	-0.6218	-0.5478
NTPC	103.14	13.9443	-0.0897	-0.1640
KOTAKBANK	1294.72	389.7079	-1.0417	0.1134
HEROMOTOCO	2767.68	380.9510	0.1448	-0.3336
HCLTECH	587.38	263.3076	1.0899	1.4329
CIPLA	625.67	156.4361	-0.0862	1.0026
SUNPHARMA	571.84	136.2263	-0.9242	0.4607
JSWSTEL	297.21	168.3382	1.0851	1.4561
UPL	514.2456	123.5668	0.3307	0.6536
DIVISLAB	1996.9506	1342.3293	-0.0778	1.0864
POWERGRID	127.8015	25.7391	2.0332	1.2335
BPCL	325.5026	60.0932	-0.3911	-0.1159
TCS	1959.8607	814.6574	-0.3063	0.7777

The securities are usually selected based on the outputs of fundamental analysis or technical analysis. These tools ensure that only the best performing stocks are included in the portfolio. No such filters are used in this study and the securities are selected at random to ensure that all types of securities are included in the portfolio. Five portfolios each with five randomly selected securities are constructed using MaxSharpe model and MaxSortino model in this study.

Portfolio-1 is constructed with ONGC, TATSTEEL, NTPC, KOTAKBANK and HEROMOTOCO and the following table gives the summary of the statistics. The variance co-variance matrices formed with excess returns and negative excess returns of the stocks included in portfolio-1 are given in Table (3) and Table (4). The MaxSharpe model and MaxSortino model are solved for the weights using Excel Solver. The Linear programming models are solved by simulating 2000 situations as specified in the models. Table (5) shows that the portfolio returns due to MaxSortino model are greater and the risk is less as compared to MaxSharpe model.

Table-2: Statistics of the securities in Portfolio-1

	ONGC	TATASTEEL	NTPC	KOTAKBK	HEROMOTO
Average Monthly return	1.18	3.05	1.06	1.88	0.61
Monthly variance	98.59	133.59	53.86	57.62	69.36
Annual return	14.18	36.64	12.70	22.52	7.34
Annual variance	1183.07	1603.08	646.34	691.39	832.26

Table-(3): Portfolio-1: Variance covariance matrix-Excess Returns

	ONGC	TATASTEEL	NTPC	KOTAKBANK	HEROMOTOCO
ONGC	1183.07	712.41	622.80	382.01	422.65
TATASTEEL	712.41	1603.08	429.86	440.70	371.36
NTPC	622.80	429.86	646.34	260.58	256.61
KOTAKBANK	382.01	440.70	260.58	691.39	234.80
HEROMOTOCO	422.65	371.36	256.61	234.80	832.26

Table-(4): Portfolio-1: Variance covariance matrix-Negative Excess Returns

	ONGC	TATASTEEL	NTPC	KOTAKBANK	HEROMOTOCO
ONGC	585.27	507.13	330.72	260.91	301.75



TATASTEEL	507.13	765.34	339.10	315.79	312.00
NTPC	330.72	339.10	294.06	195.34	201.91
KOTAKBANK	260.91	315.79	195.34	331.48	192.36
HEROMOTOCO	301.75	312.00	201.91	192.36	358.50

Stocks	MaxSharpe Model	MaxSortino Model
ONGC	0.00	0.00
TATASTEEL	0.53	0.64
NTPC	0.00	0.00
KOTAKBANK	0.47	0.36
HEROMOTOCO	0.00	0.00
sum	1.00	1.00
Expected return	29.93564	31.56164
Risk	28.59864	22.41014
RATIO	Sharpe's Ratio: 0.821215	Sortino's Ratio: 1.120548

Portfolio-2 is constructed with HCLTECH, CIPLA, SUNPHARMA, JSWSTEEL and UPL and table (6) gives the summary of the statistics.

	HCLTECH	CIPLA	SUNPHARMA	JSWSTEL	UPL
Average Monthly return	2.13	1.13	0.40	3.32	2.28
Monthly variance	58.86	63.05	78.10	136.01	105.20
Annual return	25.57	13.53	4.81	39.82	27.32
Annual variance	706.31	756.54	937.22	1632.10	1262.36

The table-7 and table-8 gives the variance co-variance matrices formed with excess returns and negative excess returns of the stocks included in portfolio-2.

	HCLTECH	CIPLA	SUNPHAR	JSWSTEL	UPL
HCLTECH	706.31	345.37	395.09	261.30	309.63
CIPLA	345.37	756.54	537.97	360.35	118.96
SUNPHAR	395.09	537.97	937.22	456.29	168.70
JSWSTEL	261.30	360.35	456.29	1632.10	524.29
UPL	309.63	118.96	168.70	524.29	1262.36

	HCLTECH	CIPLA	SUNPHAR	JSWSTEL	UPL
HCLTECH	309.72	157.98	190.24	270.33	233.31
CIPLA	157.98	284.09	231.25	173.65	103.18
SUNPHAR	190.24	231.25	407.67	241.83	136.09
JSWSTEL	270.33	173.65	241.83	693.51	447.24
UPL	233.31	103.18	136.09	447.24	588.52



The optimal solution obtained by simulating 2000 situations as specified in the models are given in Table (9). The portfolio returns is more in MaxSortino model while the risk is less as compared to MaxSharpe model.

Stocks	MaxSharpe Model	MaxSortino Model
HCLTECH	0.47	0.45
CIPLA	0.00	0.00
SUNPHARMA	0.00	0.00
JSWSTEL	0.39	0.55
UPL	0.13	0.00
sum	1.00	1.00
Expected return	31.39501	33.39372
Risk	24.98168	20.15004
RATIO	Sharpe's Ratio: 0.998532	Sortino's Ratio: 1.337155

Portfolio-3 is constructed with BAJAFINANCE, INDUSINDBANK, BHARTIARTL, GRASIM and INFY and table (10) gives the summary of the statistics.

	BAJAJFIN	INDUSIND	BHARTIARTEL	GRASIM	INFY
Average Monthly return	4.61	1.46	1.62	2.25	2.22
Monthly variance	177.52	202.61	65.13	74.07	52.04
Annual return	55.26	17.47	19.42	26.95	26.62
Annual variance	2130.29	2431.36	781.55	888.90	624.52

The table-11 and table-12 gives the variance co-variance matrices formed with excess returns and negative excess returns of the stocks included in portfolio-1.

	BAJAJFIN	INDUSIND	BHARTIARTEL	GRASIM	INFY
BAJAJFIN	2130.29	1600.67	410.07	528.80	195.56
INDUSIND	1600.67	2431.36	453.28	736.15	330.68
BHARTIARTEL	410.07	453.28	781.55	296.17	49.62
GRASIM	528.80	736.15	296.17	888.90	66.43
INFY	195.56	330.68	49.62	66.43	624.52

	BAJAJFIN	INDUSIND	BHARTIARTEL	GRASIM	INFY
BAJAJFIN	1,053.27	972.60	363.39	496.29	218.66
INDUSIND	972.60	1,359.79	378.10	597.67	288.67
BHARTIARTEL	363.39	378.10	391.76	264.78	113.37
GRASIM	496.29	597.67	264.78	539.69	172.56
INFY	218.66	288.67	113.37	172.56	272.41

The optimal weights are derived by simulating 2000 situations satisfying the conditions specified by the models. According to the results in table (13) Sortino's ratio leads to a return of 39.70 as compared to 35.64 by Sharpe's ratio. The risk due to MaxSortino is less than the risk due to MaxSharpe.



Stocks	MaxSharpe Model	MaxSortino Model
BAJAJFIN	0.32	0.46
INDUSIND	0.00	0.00
BHARTIARTEL	0.03	0.00
GRASIM	0.18	0.00
INFY	0.46	0.54
sum	1.00	1.00
Expected return	35.64368	39.70387
Risk	23.01221	20.21557
RATIO	Sharpe's Ratio: 1.268617	Sortino's Ratio: 1.644963

Portfolio-4 is constructed with BHARTIARTL, ONGC, NTPC, CIPLA, SUNPHARMA and table (14) gives the summary of the statistics.

	BHARTIARTL	ONGC	NTPC	CIPLA	SUNPHARMA
Average Monthly return	1.62	1.18	1.06	1.13	0.40
Monthly variance	65.13	98.59	53.86	63.05	78.10
Annual return	19.42	14.18	12.70	13.53	4.81
Annual variance	781.55	1183.07	646.34	756.54	937.22

The table-15 and table-16 gives the variance co-variance matrices formed with excess returns and negative excess returns of the stocks included in portfolio-4.

	BHARTIARTL	ONGC	NTPC	CIPLA	SUNPHARMA
BHARTIARTL	781.55	317.52	183.90	147.12	212.00
ONGC	317.52	1183.07	622.80	261.85	350.06
NTPC	183.90	622.80	646.34	100.07	176.54
CIPLA	147.12	261.85	100.07	756.54	537.97
SUNPHARMA	212.00	350.06	176.54	537.97	937.22

	BHARTIARTL	ONGC	NTPC	CIPLA	SUNPHARMA
BHARTIARTL	391.76	240.87	179.47	103.51	140.00
ONGC	240.87	585.27	330.72	196.62	218.67
NTPC	179.47	330.72	294.06	121.28	145.79
CIPLA	103.51	196.62	121.28	284.09	231.25
SUNPHARMA	140.00	218.67	145.79	231.25	407.67

Table (17) gives the optimal solution for the two models and it can be observed that the portfolio return is maximum in MaxSortino and the risk is minimum due to MaxSortino.

Stocks	MaxSharpe Model	MaxSortino Model
BHARTIARTL	0.32	0.46
ONGC	0.00	0.00



NTPC	0.03	0.00
CIPLA	0.18	0.00
SUNPHARMA	0.46	0.54
sum	1.00	1.00
Expected return	11.35713	11.48321
Risk	22.10664	16.4777
RATIO	Sharpe's Ratio:0.221976	Sortino's Ratio: 0.305456

Portfolio-5 is constructed with DIVISLAB, POWERGRID, NTPC, BPCL and TCS. The summary statistics are in table (18).

	DIVISLAB	POWERGRID	NTPC	BPCL	TCS
Average Monthly return	2.70	1.56	1.08	1.73	2.20
Monthly variance	80.47	33.85	54.07	117.24	50.31
Annual return	32.41	18.67	12.91	20.75	26.37
Annual variance	965.64	406.23	648.86	1406.82	603.74

The table-19 and table-20 gives the variance co-variance matrices formed with excess returns and negative excess returns of the stocks included in portfolio-5.

	DIVISLAB	POWERGRID	NTPC	BPCL	TCS
DIVISLAB	965.64	12.91	97.49	-20.29	2.81
POWERGRID	12.91	406.23	325.45	252.96	-12.39
NTPC	97.49	325.45	648.86	360.22	9.12
BPCL	-20.29	252.96	360.22	1406.82	95.84
TCS	2.81	-12.39	9.12	95.84	603.74

	DIVISLAB	OWERGRID	NTPC	BPCL	TCS
DIVISLAB	563.52	108.84	143.63	153.92	102.59
POWERGRID	108.84	182.68	165.76	197.33	78.66
NTPC	143.63	165.76	295.25	267.66	117.62
BPCL	153.92	197.33	267.66	652.45	230.18
TCS	102.59	78.66	117.62	230.18	259.04

The optimal weights obtained through simulation using Excel solver are in table (21). The solution suggests that MaxSortino performs well in terms of return and risk as compared to MaxSharpe.

Stocks	MaxSharpe Model	MaxSortino Model
DIVISLAB	0.29	0.28
POWERGRID	0.31	0.21
NTPC	0.00	0.00
BPCL	0.04	0.00
TCS	0.36	0.51
sum	1.00	1.00



Expected return	25.5368566	26.41737
Risk	14.4932193	13.34504
Ratio	Sharpe's Ratio:1.31695079	Sortino's Ratio: 1.496239

4. CONCLUSION:

Table (22) summarizes the findings with a highlight of the returns and risk due to the two competing models used in this study. The study started with a question that considering the positive deviations while computing the risk may underestimate the portfolio. With this doubt, the Sortino's ratio which uses the variance co-variance matrix of negative excess returns to calculate the portfolio risk is used as an objective in the MaxSortino model. The findings suggest that MaxSortino model outperforms MaxSharpe model as the returns of this model is greater and the risk is comparatively less. Hence the study concludes that in the best objective while constructing an efficient portfolio is to maximize the Sortino's ratio rather than the Sharpe's ratio.

Table (22): Portfolio Returns and Risk-MaxSharpe and MaxSortino models

Portfolio	MaxSharpe Model		MaxSortino Model	
	Expected Return	Risk	Expected Return	Risk
1	29.94	28.60	31.56	22.41
2	31.40	24.98	33.39	20.15
3	35.64	23.01	39.70	20.21
4	11.35	22.10	11.48	16.48
5	25.54	14.49	26.41	13.35

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