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Symmetry and Topology: The 11 Uninodal PlanarNets Revisited

J. USHA
Assistant Professor, Department of Mathematics,
St.Antony's College of Arts and Sciences for Women, Thamaraipadi, Dindigul
Email – ignatiususha@gmail.com

Abstract: *A depiction of the 11 notable uninodal planar nets is given by Cayley variety diagrams or option Cayley variety charts of plane gatherings. By applying techniques from topological diagram hypothesis, the nets are gotten from the bouquet Bn with turns for the most part as voltages. It hence gives the idea that interpretation, as an evenness activity in these nets, is not any more central than revolutions.*

Key Words: *uninodal nets; crystal structures; symmetry; labeled quotient graphs.*

1. INTRODUCTION:

The paper is coordinated as follows. Definitions and principal properties of voltage diagrams and their application to the depiction of the combinatorial geography of precious stone designs are checked on in Segment 2. The primary text manages the utilization of the philosophy to the portrayal of uninodal two-intermittent (planar) nets. These nets were picked due to their significance in precious stone science [7] as well concerning their scant number: it is realized that there are precisely 11 uninodal two-occasional planar nets [8]. Segment 3 presents a portrayal of the square grid net sql as per the vector-technique, including an investigation of the ideal two-layered space-bunch (plane gathering) of the net. Segment 4 presents a portrayal of sql utilizing an evenness marked remainder diagram, showing that the full plane gathering of the net, including its translational balance, is produced by two legitimate pivots. The examination of the 10 leftover uninodal two-occasional planar nets is performed similarly in the following segments; the various nets are dissected arranged by developing intricacy. The paper closes for certain broad perceptions concerning the depiction of uninodal planar nets.

2. METHODOLOGY:

The technique is adjusted from the work created by Gross and Exhaust [3]. Voltage diagrams applied to the examination of gem structures are chart hypothetical items that can be casually deciphered as a combinatorial depiction of the deviated unit of an intermittent design. Nonetheless, a voltage chart likewise conveys data about the evenness tasks that are important to reproduce the entire construction from the hilter kilter unit, which makes them so strong. Since here we confine our examination to uninodal nets, the unbalanced unit contains a solitary vertex; for this situation, all edges of the voltage diagram start and end at a similar vertex. Such edges are called circles and the relating diagram is known as a bouquet; the bouquet B_π concedes n circles. These circles are situated and relegated an evenness activity, called the voltage on the circle. A remarkable construction can then be gotten from this voltage diagram as follows. To begin with, we structure the balance bunch G produced by the entire arrangement of voltages. We call V the single vertex of the bouquet B_π; the vertex set of the inferred chart is characterized as the set $\{V_g: g \in G\}$. Voltages show which vertices of the determined diagram must be connected. We assume some circle is relegated voltage σ. Then, at that point, for each $g \in G$, there is an edge beginning at vertex V_g and finishing at vertex $V_{g\sigma}$. This edge, normally signified as σ_g, is a situated edge. From [3], it is realized that the gathering G acts uninhibitedly on the determined diagram as follows: An evenness activity $f \in G$ maps (I) vertex V_g to vertex V_{fg}, and (ii) edge $\sigma_g = V_g V_{g\sigma}$ to edge $\sigma_{fg} = V_{fg} V_{fg\sigma}$. Subsequently, G is a subgroup of the full balance gathering of the inferred diagram. The chart got from the past development is, truth be told known as the Cayley variety diagram of the gathering G with voltages as generators (colors). It might happen that Cayley variety diagrams present twofold edges when the individual generator has request 2. To be sure, assuming $\sigma_2 = 1$, then the two edges σ_g and $\sigma_{g\sigma}$, which are different by development, connect the two vertices V_g and $V_{g\sigma}$. For this situation, we substitute the sets of arranged edges by a solitary non-situated edge. This changed development is known as the option Cayley variety chart of the gathering. Obviously, precious stone designs bear no direction, yet it very well might be useful to keep directions in the determined diagram to clarify the relationship with the voltage chart. A corresponded approach is utilized in [9], where the creators give

various portrayals of the 17 two-layered space-gatherings (plane gatherings) through a rundown of potential generators and related relators. They have likewise drawn the individual Cayley outlines (Cayley and elective Cayley charts), which are normally isomorphic to uninodal two-intermittent planar nets. Be that as it may, a few of the created nets end up being isomorphic, and, on the other hand, not all planar nets have been determined. The four nets cem, fsz, htb and tts are absent. This paper centers around planar nets, finding out if they can be generally gotten from the bouquet Bn only involving point-balance tasks as voltages. The proposed portrayal likewise follows a guideline of economy, searching for the littlest conceivable number of circles in the bouquet. Obviously, at least two circles is important to depict a two-occasional net, whatever the idea of the voltage. The outcomes revealed in Table 1 demonstrate the way that most planar nets can be gotten from B_2 or B_3.

The Square Lattice Net:

The Vector Method: As a delineation of the overall strategy, we consider the depiction of the sql net from its named remainder diagram, the bouquet B_2, as given in Figure 1. For this situation, as per the vector technique [4], voltages are vectors in Z_2, and the produced bunch is an interpretation gathering of rank 2. The vertex set is characterized as $\{V_t : t \in Z_2\}$. If two symmetrical vectors a and b are utilized as a premise of the grid in the plane, with correspondence a = 10 and b = 01, one gets the determined net with the direction as given in Figure 1. That's what we see, locally, the inferred net has a similar construction as the voltage chart: for each tone, there is one active and one approaching edge at each vertex. The green edge at joins vertex V_t to vertex V_{t+a}, and the red edge b_t joins vertex V_t to vertex V_{t+b}.

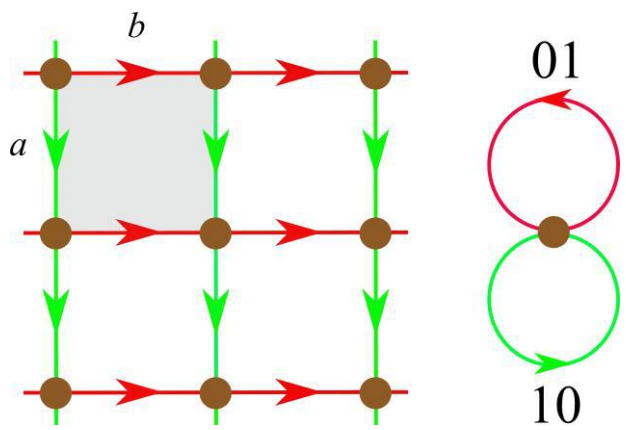

Figure1

Fig.1 - (Left) A mathematical acknowledgment (implanting) of the square grid net (sql) in the Euclidian plane with situated edges, and (Right) its marked remainder diagram with voltages in Z_2. The two classes of edges in the net are given a similar variety as the delegate circle in B_2.

We consider now the full evenness gathering of the sql net. Since sql is a grid net, it is a crystallographic net [10], and that implies, by definition, that its automorphism bunch is isomorphic to a space-bunch [11]. Since it is likewise an insignificant net [12], the component gathering of its space-bunch is isomorphic to the automorphism gathering of the voltage diagram [2]. To decide the point gathering of sql, we accordingly search for generators of the automorphism gathering of the bouquet B_2 and afterward for an understanding as evenness tasks in Euclidian space. These numerical properties emerge from the connection between circles in the bouquet and lines in the implanting of the net. Because of the development technique portrayed over, a circle opens up along an endless line situated in the crystallographic course given by the related voltage; the relationship is confirmed in Fig.1 through both the variety and the direction of the particular components. For example, the red circle with voltage 01 opens up along red lines with heading 01. Comparably, any red (green) line projects onto the red (green) circle. The impact of a point evenness procedure on the net is to play out a stage of the circles in the voltage diagram. For example, the appearance in the a-pivot changes the direction of each and every line along b; as a stage, it tends to be composed (b, −b). The automorphism gathering of the bouquet B_2 is produced by the three changes (a, −a), (b, −b) and (a, b) related separately to appearance in the axes (0, x), (x, 0) and (x, x) and is hence of request 8. Since there is a solitary vertex for every unit cell, the greatest space-gathering of sql is the symmorphic bunch p4mm.

An Example of a Symmetry-Labeled Quotient Graph: The above description of **sql** is based on translation operations in the Euclidian plane. In this section, we consider extensively a derivation of **sql** from the bouquet B_2 with two 4-fold rotations as voltages on the loops, as shown in Figure 2. The given representation of the net was obtained after placing the initial vertex close to the origin and considering two anticlockwise 4-fold rotations α and β with centers at (1/2,

−1/2) and (−1/2, 1/2), respectively. These initial elements are shown in brown in the figure. Because voltages have order 4, each loop unwraps to a 4-cycle: starting from vertex V_1 at the origin, the green loop unwraps to the green 4-cycle around the center of rotation β, and similarlythe red loop unwraps to the red 4-cycle around the center of rotation α. Because we know that β acts freely on the derived net, we also obtain by this rotation the three other red 4-cycles at the corners of the unit cell. We note that this unit cell, as drawn in Figure 2, corresponds to the space-group generated by the two rotations α and β, which happens to be a 2 × 2 supercell of **sql**.

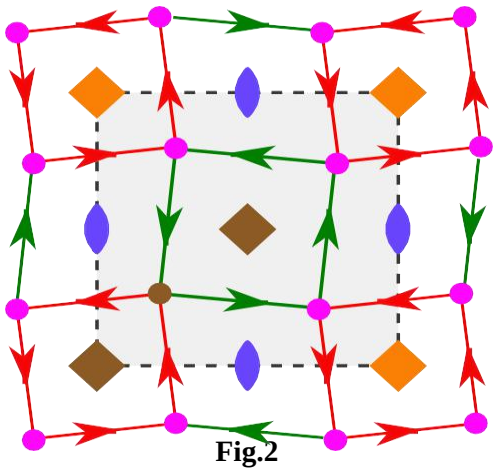

Fig.2

Fig.2 (Left) The square grid net and an emblematic portrayal of the space-bunch produced by a solitary vertex and two 4-overlay revolution focuses (in brown) utilizing (Right) the bouquet B_2 with the particular pivots α and β as voltages. The two classes of edges in the net are given a similar variety as the delegate circle in B_2. Note that the underlying vertex has been marginally moved comparable to the beginning, in this manner furnishing a genuinely p4 implanting with a 2 × 2 unit cell. To obtain a better understanding of the derived net, or equivalently of the Cayley color graph ofthe generated group, we can use a representation of the two initial rotations by extended 3 × 3 matricessuch as follows:

$$\alpha = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 0 & 1 \end{bmatrix} \quad \beta = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

We note first that the two combinations $\alpha\beta$ and $\beta\alpha$ represent 2-fold rotations with centers at (−1/2, −1/2) and (1/2, 1/2), respectively.

$$\alpha\beta = \begin{bmatrix} -1 & 0 & -1 \\ 0 & -1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \quad \beta\alpha = \begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

We also note that monomials of degree 4 correspond to translations, such as the twofollowing combinations.

$$\alpha^3\beta = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \beta\alpha^3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

More generally, the two generators satisfy the three relations $\alpha^4 = \beta^4 = (\alpha\beta)^2 = 1$. From an abstract point of view, these relations are enough to define the group $G = \langle\alpha, \beta\rangle$. It can be checked that the group

$T = \langle\alpha^3\beta, \beta\alpha^3\rangle$, generated by the two given translations, is abelian and normal in $G = \langle\alpha, \beta\rangle$ and that the factor group G/T admits four cosets represented by 1, β, β and β , showing that G, as an abstract group, is isomorphic to the space-group $p4$, but with a 2 × 2 unit cell in comparison with the primitive cell of the former embedding shown in Figure 1. The identification of the derived net N with **sql** can be achieved by analyzing the labeled quotient graph N/T, as follows.
It is more appropriate in this case to construct N/T directly from N/G instead of using the derived net N. This can be done by denoting the four vertex cosets as $T\beta^n$ with $n \in \{0, 1, 2, 3\}$ and working out the edges between them as well as the respective voltages in T. We consider first the loop with voltage β in N/G: this loop indicates that, for any $t \in T$, there

∈

are two edges linking vertex $t\beta^n$ to vertices $t\beta^{n-1}$ and $t\beta^{n+1}$. We thus have four edges forming a 4-cycle with zero voltage in N/T. The case of the loop with voltage α is more difficult. There are similarly two edges from $t\beta^n$ to $t\beta^n\alpha$ and $t\beta^n\alpha^3$; these vertices should be first rewritten as $tx\beta^m$ with x T in order to assign voltage x to the edge from $t\beta^n$ to $t\beta^m$ in N/T. For instance,

$$\beta\alpha = x\beta^m \quad \Rightarrow \quad \{m = 2 \quad \text{and} \quad x = \beta\alpha\beta^2 = \alpha^3\beta\} \qquad (1)$$

The value $m = 2$ is chosen in order to obtain a monomial of degree 4 for the translation x. The finalresult comes from the relation $\beta\alpha\beta = \alpha^3$ in G (a consequence of the relations $\alpha^4 = (\alpha\beta)^2 = 1$). We should thus add an edge from $T\beta$ to $T\beta^2$ with voltage $\alpha^3\beta$.

The complete labeled quotient graph is [11], where the two translations $\alpha^3\beta$ and $\beta\alpha^3$ have been written as 20 and 02, respectively,in accordance with the above matrices.The isomorphism between the derived net N and **sql** can be worked out through labeled quotient graphs, as indicated [7]. There are indeed two freely acting automorphisms of N/T that leave the voltages over cycles unchanged and that should be interpreted as images of translations in N,thus extending the group T. We first define the automorphism

$\theta_v = (T, T\beta)(T\beta^2, T\beta^3)$ exchanging (i) T with $T\beta$ and $T\beta^2$ with $T\beta^3$; (ii) green and red *horizontal* edges between T and $T\beta$, as well as those between $T\beta^2$ and $T\beta^3$; and (iii) *vertical* edges between T and $T\beta^3$ with those of the same colorbetween $T\beta$ and $T\beta^2$. A second automorphism $\theta_h = (T, T\beta^3)(T\beta, T\beta^2)$ is defined similarly, this time exchanging colors for *vertical* edges and keeping colors for *horizontal* edges; θ_v and θ_h act on N/T as reflections in the blue v and h lines, respectively. The graph N/T thus has a single vertex class and two edge classes for the automorphism group $\langle\theta_h, \theta_v\rangle$; the four *horizontal* edges form a first class and the four *vertical* edges form a second class. Hence its quotient is the bouquet B_2 with all horizontal edges mapped on the upper loop and all vertical edges mapped on the lower loop. More precisely, both 2-cycles with voltage 02 (resp. 20) are wrapped on the upper (resp. lower) loop. This means that the voltages on the loops are respectively 01 and 10.

4. CONCLUSION:

In this paper, We discussed about methodology, the square lattice net and the vector method with an example of a symmetry-labeled quotient graph. The use of symmetry-labeled quotient graphs as representations of periodic nets presents some advantages compared with the more usual description by translation-labeled quotient graphs.

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