



On Nano Contra Regular \wedge Generalized Continuous functions in Nano Topological Spaces

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Abstract: In this paper we introduce and investigate a new class of continuous functions called Nano Contra $R^{\wedge}G$ -continuous functions. Furthermore we introduce Almost Nano Contra $R^{\wedge}G$ -continuous functions and investigate several properties of the new notions.

Key Words: Contra $R^{\wedge}G$ -continuous function, Almost Nano Contra $R^{\wedge}G$ -continuous function.

1. INTRODUCTION:

The theory of nano topology was introduced by Lellis Thivagar et al. [18]. They defined a nano topological space with respect to a subset X of a universe U which is defined based on lower and upper approximations of X and also make known about nano-closed sets, nano-interior, nano closure and weak form of nano open sets namely nano semi-open sets, nano pre-open, nano α -open sets and nano β -open sets. Nasef et al. make known about some of nearly open sets in nano topological spaces. Revathy and Gnanambal Ilango gave the idea about the nano -open sets. Sathishmohan et al. bring up the idea about nano neighbourhoods in nano topological spaces. And also Mythili Gnanapriya [13] introduced Nano generalized closed set and nano generalized continuous functions and studied their properties.

Different type of generalizations of continuous functions was studied various authors in the recent development of topology. In 1991, Balachandran et al [3], introduced and studied the notions of generalized continuous functions. The concept of Regular \wedge generalized continuous function was introduced by Savithiri.D and Janaki.C, [22] D. Savithiri and Stephy S [23] introduced the concept of Nano $R^{\wedge}G$ closed sets in nano topological spaces

2. PRELIMINARIES:

All through this paper, X, Y, Z stand for nano topological spaces $(X, \tau), (Y, \sigma), (Z, \eta)$ with no separation axioms assumed. Let $A \subseteq X$, the closure and interior of A will be denoted by $Cl(A)$ and $Int(A)$ respectively.

Definition 2.1:

Let U be a non-empty finite set of objects called the universe R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.

(i) The Lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $LR(X)$.

That is, $LR(X) = \{U\{R(x) : R(x) \subseteq X\}\}$; where $R(x)$ denotes the equivalence class determined by x .

(ii) The Upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by

$UR(X) = \{U\{R(x) : R(x) \cap X \neq \emptyset\}\}$

(iii) The Boundary region of X with respect to R is the set of all objects which can be classified neither as X nor as $\neg X$ with respect to R and it is denoted by

$BR(X) = UR(X) - LR(X)$.

Definition 2.2:

Let U be the universe, R be an equivalence relation on U and $\tau R(X) = \{U, \emptyset, LR(X), UR(X), BR(X)\}$ where $X \subseteq U$ and $\tau R(X)$ satisfies the following axioms.



(i) U and $\emptyset \in \tau_R(X)$.

(ii) The union of the elements of any sub collection $\tau_R(X)$ is in $\tau_R(X)$.

(iii) The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

That is, $\tau_R(X)$ forms a topology U called as the nano topology on U with respect to X . We call $(U, \tau_R(X))$ as the nano topological space. The elements of $\tau_R(X)$ are called as nano open sets. A set A is said to be nano closed if its complement is nano open.

Definition 2.3: A nano subset A of X is called nano $r^{\wedge}g$ closed [22] if $Ngcl(A) \subseteq U$ whenever $A \subseteq U$ and U is nano regular open in X .

Definition 2.4: A nano subset A of X is called nano g - closed set if $Ncl(A) \subseteq U$ whenever $A \subseteq U$ and U is nano open in X .

Definition 2.5: A nano subset A of X is called nano g^* closed set if $Ncl(A) \subseteq U$ whenever $A \subseteq U$ and U is nano g open in X .

Definition 2.6: A nano subset A of X is called nano rgw -closed set if $Ncl(Nint(A)) \subseteq U$ whenever $A \subseteq U$ and U is nano regular open in X .

Definition 2.7: A nano subset A of X is nano rwg closed [18] set if $Ncl(Nint(A)) \subseteq U$ whenever $A \subseteq U$ and U is nano regular semi open in X .

Definition 2.8: A nano subset A of X is called a nano regular closed [18] set if $A = Ncl(Nint(A))$.

Definition 2.9: A nano subset A of X is called regular open in x if $A = Nint(Ncl(A))$.

Definition 2.10: A nano subset A of X is called a nano semi-open set if $A \subseteq Ncl(Nint(A))$;

Definition 2.11: A nano subset A of X is called nano semi closed set if $Nint(Ncl(A)) \subseteq A$.

Definition 2.12: A nano subset A is called nano regular open [9] if $A = Nint(Ncl(A))$ and its complement is called nano regular closed. A nano subset A is called nano g closed [9] if $Ncl(A) \subseteq U$ whenever $A \subseteq U$ and U is nano open. The intersection of all nano g closed sets is called nano g -closure of A and it is denoted by $Ngcl(A)$.

Definition 2.13: A map $f: (X, \tau) \rightarrow (Y, \sigma)$ is called nano $r^{\wedge}g$ - continuous [22] if $f^{-1}(V)$ is nano $r^{\wedge}g$ closed in X for every nano closed set V in Y .

Definition 2.14: A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is called nano contra continuous [2] if $f^{-1}(V)$ is nano closed in X for each nano open set V of Y .

Definition 2.15: A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is called nano contra g -continuous [9] iff $f^{-1}(V)$ is nano g -closed in X for each nano open set V of Y .

Definition 2.16: A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is called nano contra g^* -continuous [4] if $f^{-1}(V)$ is nano g^* -closed in X for each nano open set V of Y .

Definition 2.17: A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is called nano contra rwg -continuous [4] iff $f^{-1}(V)$ is nanorwg-closed in X for each nano open set V of Y .

Definition 2.18: A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is called nano contra rgw -continuous [5] if $f^{-1}(V)$ is nano rgw -closed in X for each nano open set V of Y .

Definition 2.19:

A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is called nano contra pr -continuous if [4] $f^{-1}(V)$ is nano pr -closed in X for each nano open set V of Y .

Definition 2.20: A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is called nano RC – continuous [4] if $f^{-1}(V)$ is nano regular closed in X for each nano open set V of Y .

Definition 2.21: A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is called nano perfectly continuous [4] if $f^{-1}(V)$ is nano clopen in X for every nano open set V of Y .

Definition 2.22: A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is called an nano R - map [4] if $f^{-1}(V)$ is nano regular closed in X for every nano regular closed set V of Y .

Definition 2.23: A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is called a nano contra R - map [4] if $f^{-1}(V)$ is nano regular closed in X for every nano regular open set V of Y .

Definition 2.24: A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is called almost nano continuous [1] if $f^{-1}(V)$ is nano closed in X for every nano regular closed set V of Y .

Definition 2.25: A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be almost nano contra continuous if $f^{-1}(V)$ is nano closed in X for each nano regular open set V in Y .

Definition 2.26: Let A be a nano subset of (X, τ) then the set $\bigcap \{U \in \tau, A \subseteq U\}$ is called the nano kernel of A and it is denoted by $\ker(A)$.

Definition 2.27: A space (X, τ) is

i) $T^{1/2}$ space [9] if every nano $r^{\wedge}g$ - closed set is nano g -closed.



ii) Locally indiscrete[2] if every nano open subset of X is nano closed.

3. NANO CONTRA $R^{\wedge}G$ - CONTINUOUS FUNCTIONS:

Definition 3.1: A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is called nano contra $r^{\wedge}g$ -continuous if the pre image of every nano open subset of Y is nano $r^{\wedge}g$ -closed in X .

Definition 3.2: A space (X, τ) is nano $r^{\wedge}g$ -locally indiscrete if every nano $r^{\wedge}g$ -open set is nano closed.

Theorem 3.3: Every nano contra continuous, nano contra g -continuous, nano contra g^* -continuous function is nano contra $r^{\wedge}g$ -continuous.

Proof: Straight forward.

Remark 3.4: The converse of the above theorem need not be true as seen in the following example.

Example 3.5: Let $X = \{a, b, c, d\}$, $Y = \{p, q, r, s\}$, $\tau = \{\emptyset, X, \{a, b\}\}$, $\sigma = \{\emptyset, Y, \{p, q, r\}, \{p, q\}, \{r\}\}$. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ defined by $f(a)=r, f(b)=s, f(c)=q$ and $f(d)=p$. Then f is nano contra $r^{\wedge}g$ -continuous but it is not a nano contra continuous, not a nano contra g -continuous and it is not a nano contra g^* -continuous function.

Theorem 3.6: Every nano RC-continuous, nano $r^{\wedge}g$ continuous function is nano contra $r^{\wedge}g$ -continuous functions.

Proof: Straight forward.

Remark 3.7: The converse of the above theorem need not be true as shown in the following example.

Example 3.8: Let $X = \{1, 2, 3, 4, 5\}$, $Y = \{a, b, c, d, e\}$, $\tau = \{\emptyset, X, \{1\}, \{1, 2\}, \{2\}\}$, $\sigma = \{\emptyset, Y, \{a, b, c\}, \{a, b, c, d\}, \{d\}\}$ and $f: (X, \tau) \rightarrow (Y, \sigma)$ be a function defined by $f(1) = d, f(2) = e, f(3) = a, f(4) = b, f(5) = c$ is nano contra $r^{\wedge}g$ -continuous but it is not nano RC-continuous.

i) Let $X = \{a, b, c, d, e\}$, $Y = \{p, q, r, s, t\}$, $\tau = \{\emptyset, X, \{a, b, c\}, \{a, b, c, d\}, \{d\}\}$, $\sigma = \{\emptyset, Y, \{q, r, s\}\}$ and $f: (X, \tau) \rightarrow (Y, \sigma)$ defined by $f(a) = p, f(b) = q, f(c) = t, f(d) = r, f(e) = s$ is nano contra $r^{\wedge}g$ -continuous but it is not nano $r^{\wedge}g$ -continuous.

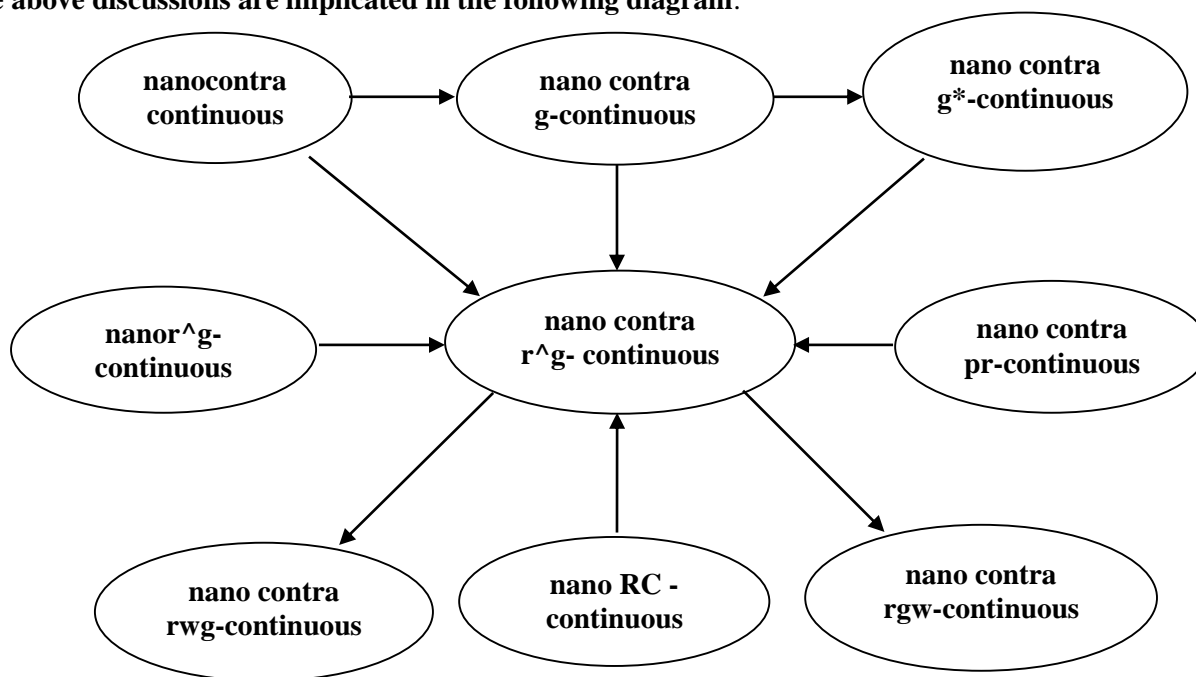
Theorem 3.9: Every nano contra $r^{\wedge}g$ -continuous function is nano contra rwg -continuous, nano contra rgw continuous, nano contra pr -continuous functions.

Proof: Straight forward.

Remark 3.10: The converse of the above theorem need not be true as shown in the following example.

Example 3.11: Let $X = \{a, b, c, d, e\}$, $Y = \{1, 2, 3, 4, 5\}$, $\tau = \{\emptyset, X, \{a, b, c\}, \{a, b, c, d\}, \{d\}\}$, $\sigma = \{\emptyset, Y, \{1\}, \{1, 2\}, \{2\}\}$ and $f: (X, \tau) \rightarrow (Y, \sigma)$ be a function defined by $f(a) = 2, f(b) = 1, f(c) = 4, f(d) = 3, f(e) = 5$ is nano contra rwg -continuous, nano contra rgw -continuous, and nano pr -continuous but it is not a nano contra $r^{\wedge}g$ -continuous function.

The above discussions are implicated in the following diagram.



Theorem 3.12:

Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a function,



- i) If f is nano $r^{\wedge}g$ -continuous and X is nano $r^{\wedge}g$ -locally indiscrete then f is nano contra continuous.
 ii) If f is nano contra $r^{\wedge}g$ -continuous and X is $T^{1/2}$ space then f is nano contra g -continuous.

Proof:

- (i) Suppose that f is nano $r^{\wedge}g$ -continuous. Let X be nano $r^{\wedge}g$ -locally indiscrete and let V be nano open in Y . Since f is nano $r^{\wedge}g$ -continuous, $f^{-1}(V)$ is nano $r^{\wedge}g$ -open in X . By hypothesis $f^{-1}(V)$ is nano closed in X . Hence f is nano contra continuous.
 (ii) Let f be nano contra $r^{\wedge}g$ -continuous and X is $T^{1/2}$ space. Let V be nano open subset of Y . By hypothesis, $f^{-1}(V)$ is nano $r^{\wedge}g$ -closed in X . Since X is $T^{1/2}$ space, every nano $r^{\wedge}g$ -closed set is nano g -closed in X . Hence f is nano contra g -continuous.

Theorem 3.13: For a function $f: (X, \tau) \rightarrow (Y, \sigma)$, the following conditions are equivalent.

- (a) f is nano contra $r^{\wedge}g$ -continuous.
 (b) For every nano closed subsets F of Y $f^{-1}(F) \in R^{\wedge}GO(X, x)$.
 (c) For each $x \in X$ and each $f \in C(Y, f(x))$, there exists $U \in R^{\wedge}GO(X, x)$ such that $f(U) \in F$.
 (d) $f(r^{\wedge}g \text{ Cl}(A)) \subset \ker(f(A))$ for every nano subset A of X .
 (e) $r^{\wedge}gcl(f^{-1}(B)) \subset f^{-1}(\ker(B))$ for every nano subset B of Y .

Proof:

The implications (a) \rightarrow (b) and (b) \rightarrow (c) are obvious.

(c) \rightarrow (b) : Let F be any nano closed set of Y and $x \in f^{-1}(F)$. Then $f(x) \in F$ and there exists $U_x \in R^{\wedge}GO(X, x)$ such that $f(U_x) \subset F$. Therefore we obtain $f^{-1}(F) = \cup \{U_x; x \in f^{-1}(F)\}$ which is nano $r^{\wedge}g$ -open in X .

(c) \rightarrow (d) : Let A be any nano subset of X . Suppose that $y \notin \ker(f(A))$. Then by hypothesis $f \in C(Y, y)$ such that $f(A) \cap F = \emptyset$. Thus we have $A \cap f^{-1}(F) = \emptyset$ and $Nr^{\wedge}gcl(A) \cap f^{-1}(F) = \emptyset$. Therefore we obtain $f(Nr^{\wedge}gcl(A) \cap F) = \emptyset$ and $y \notin f(Nr^{\wedge}gcl(A))$. This implies that $f(Nr^{\wedge}gcl(A)) \subset \ker(f(A))$.

(d) \rightarrow (e) : Let B be any nano subset of Y . By (d), $f(Nr^{\wedge}gcl(f^{-1}(B))) \subset \ker(f(f^{-1}(B))) \subset \ker(B)$ and $(Nr^{\wedge}gcl(f^{-1}(B))) \subset f^{-1}(\ker(B))$.

(e) \rightarrow (a) : Let V be any nano open set of Y . By hypothesis, $Nr^{\wedge}gcl(f^{-1}(V)) \subset f^{-1}(\ker(V)) = f^{-1}(V)$ and $Nr^{\wedge}gcl(f^{-1}(V)) = f^{-1}(V)$. This shows that $f^{-1}(V)$ is nano $r^{\wedge}g$ closed in X . Hence f is nano contra $r^{\wedge}g$ -continuous.

Theorem 3.14: If f is nano contra $r^{\wedge}g$ -continuous and Y is nano regular, then f is nano $r^{\wedge}g$ -continuous.

Proof: Let x be an arbitrary point of X and V be a nano open set of Y containing $f(x)$. Since Y is nano regular, there exists a nano open set G in Y containing $f(x)$ such that $Ncl(G) \subseteq V$. since f is nano contra $r^{\wedge}g$ -continuous, by theorem 3.13, there exists $U \in Nr^{\wedge}gO(X, x)$ such that $f(U) \subseteq Ncl(G)$. Then $f(U) \subseteq Ncl(G) \subseteq V$. Hence f is nano $r^{\wedge}g$ -continuous.

4. ALMOST NANO CONTRA $R^{\wedge}G$ - CONTINUOUS FUNCTIONS:

Definition 4.1: A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be almost nano contra $r^{\wedge}g$ -continuous if $f^{-1}(V)$ is nano $r^{\wedge}g$ -closed in X for each nano regular open set V in Y .

Theorem 4.2: If a function $f: (X, \tau) \rightarrow (Y, \sigma)$ is almost nano contra continuous then it is almost nano contra $r^{\wedge}g$ -continuous but not conversely.

Proof: Obvious.

Example 4.3: Let $X = \{a, b, c, d, e\}$, $Y = \{1, 2, 3, 4, 5\}$, $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b, c\}, \{a, b, c, d\}, \{d\}\}$, $\sigma = \{Y, \emptyset, \{1\}, \{1, 2\}, \{2\}\}$ and $f: (X, \tau) \rightarrow (Y, \sigma)$ be a function defined by $f(a) = 3$, $f(b) = 4$, $f(c) = 5$, $f(d) = 1$, $f(e) = 2$ is almost nano contra $r^{\wedge}g$ -continuous but it is not almost nano contra continuous,

Theorem 4.4: Every nano contra $r^{\wedge}g$ -continuous function is almost nano contra $r^{\wedge}g$ -continuous function but not conversely.

Proof: Obvious.

Example 4.5: Let $X = \{a, b, c, d, e\}$, $Y = \{p, q, r, s, t\}$, $\tau = \{X, \emptyset, \{a, b, c\}, \{a, b, c, d\}, \{d\}\}$ and $\sigma = \{Y, \emptyset, \{q, r, s\}\}$. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a function defined by $f(a) = q$, $f(b) = r$, $f(c) = s$, $f(d) = p$, $f(e) = t$ is almost nano contra $r^{\wedge}g$ continuous but it is not nano contra $r^{\wedge}g$ -continuous function.

Theorem 4.6: Suppose $Nr^{\wedge}GO(X, \tau)$ is nano closed under arbitrary union. The following statements are equivalent for the function $f: (X, \tau) \rightarrow (Y, \sigma)$

- i) f is almost nano contra $r^{\wedge}g$ -continuous



- ii) for every nano regular closed set F of Y , $f^{-1}(F)$ is a nano $r^{\wedge}g$ -open set of Y .
- iii) for each $x \in X$ and each nano regular closed set F in Y containing $f(x)$, there exists a nano $r^{\wedge}g$ -open set U in X containing x such that $f^{-1}(U) \subset F$.
- iv) For each $x \in X$ and each nano regular open set V in Y not containing $f(x)$, there exists an nano $r^{\wedge}g$ -closed set K in X not containing x such that $f^{-1}(V) \subset K$.
- v) $f^{-1}(\text{Nint}(\text{Ncl}(G))) \in \text{NR}^{\wedge}\text{GC}(X, \tau)$ for every nano open subset G of Y .
- vi) $f^{-1}(\text{Nint}(\text{Ncl}(F))) \in \text{NR}^{\wedge}\text{GO}(X, \tau)$ for every nano closed subset F of Y .

Proof:(i) \Rightarrow (ii):

Let F be a nano regular closed set in Y . then $Y-F$ is a nano regular open in Y . By hypothesis, $f^{-1}(Y-F) = X - f^{-1}(F)$ is nano $r^{\wedge}g$ -closed in X . Therefore $f^{-1}(F)$ is nano $r^{\wedge}g$ -open in X .

(ii) \Rightarrow (i):

Let V be a nano regular open set in Y . Then $Y-V$ is nano regular closed in Y , by (ii) $f^{-1}(Y-V)$ is nano $r^{\wedge}g$ -open set in X . This implies $X - f^{-1}(V)$ is nano $r^{\wedge}g$ open which implies $f^{-1}(V)$ is nano $r^{\wedge}g$ -closed in X . Thus (i) holds.

(ii) \Rightarrow (iii): Let F be any nano regular closed set containing $f(x)$. then $f^{-1}(F) \in \text{NR}^{\wedge}\text{GO}(X, \tau)$ and $x \in f^{-1}(F)$ (by (ii)). By taking $U = f^{-1}(F)$ $f(U) \subset F$.

(iii) \Rightarrow (ii): Let $F \in \text{NRC}(Y, \sigma)$ and $x \in f^{-1}(F)$. from (iii), there exists an nano $r^{\wedge}g$ -open set U in X containing x such that $U \subset f^{-1}(F)$ we have $f^{-1}(F) = \cup \{ U : x \in f^{-1}(F) \}$. hence $f^{-1}(F)$ is nano $r^{\wedge}g$ -open.

(iii) \Rightarrow (iv): Let V be any nano regular open set in Y containing $f(x)$. Then $Y-V$ is a nano regular closed set containing $f(x)$. By (iii), there exists an nano $r^{\wedge}g$ -open set U in X containing x such that $f(U) \subset Y-V$. Hence $U \subset f^{-1}(Y-V) \subset X - f^{-1}(V)$. Then $f^{-1}(V) \subset X - U$. Let $X-U = K$. Thus we obtain an nano $r^{\wedge}g$ -closed set in X not containing x such that $f^{-1}(V) \subset K$.

(iv) \Rightarrow (iii): Let F be a nano regular closed set in Y containing $f(x)$. Then $Y-F$ is nano regular open set in Y containing $f(x)$. By (iv), there exists an nano $r^{\wedge}g$ -closed set K in X not containing x such that $f^{-1}(Y-F) \subset K$. Then $X - f^{-1}(F) \subset K$ implies $X-K \subset f^{-1}(F)$. Hence $f(X-K) \subset F$. Take $U = X-K$. Then U is an nano $r^{\wedge}g$ -open set in X containing x such that $f(U) \subset F$.

(v) \Rightarrow (i): Let $V \in \text{NRO}(Y, \sigma)$. Then V is nano open in Y . By (v), $f^{-1}(\text{Nint}(\text{Ncl}(G))) \in \text{NR}^{\wedge}\text{GC}(X, \tau)$ which implies $f^{-1}(V) \in \text{NR}^{\wedge}\text{GC}(X, \tau) \Rightarrow f$ is almost nano contra $r^{\wedge}g$ -continuous.

(i) \Rightarrow (v): Let G be nano open subset of Y . Since $\text{Nint}(\text{Ncl}(G))$ is nano regular open, then by (i), $f^{-1}(\text{Nint}(\text{Ncl}(G))) \in \text{NR}^{\wedge}\text{GO}(X, \tau)$.

(ii) \Leftrightarrow (vi) is as similar as **(i) \Leftrightarrow (v)**.

Theorem 4.7:

For two functions $f: (X, \tau) \rightarrow (Y, \sigma)$ and $g: (Y, \sigma) \rightarrow (Z, \eta)$, let $g \circ f: (X, \tau) \rightarrow (Z, \eta)$ is a composite function. Then the following holds.

- (i) If f is nano $r^{\wedge}g$ continuous and g is perfectly continuous then $g \circ f$ is almost nano contra $r^{\wedge}g$ -continuous.
- (ii) If f is nano contra $r^{\wedge}g$ -continuous and g is almost nano continuous then $g \circ f$ is almost nano contra $r^{\wedge}g$ -continuous.
- (iii) If f is nanor $^{\wedge}g$ -continuous and g is almost nano contra g -continuous and Y is $T_{1/2}$ space then $g \circ f$ is almost nano contra $r^{\wedge}g$ -continuous.

Proof: (i) Let V be a nano regular open set of Z . Every nano regular open set is nano open and g is perfectly continuous, therefore $g^{-1}(V)$ is nano closed in Y . Since f is nanor $^{\wedge}g$ -continuous. $f^{-1}(g^{-1}(V))$ is nano $r^{\wedge}g$ -closed in X . Thus $g \circ f$ is almost nano contra $r^{\wedge}g$ -continuous.

(ii) Let V be a nano regular open set of Z . Since g is almost nano continuous then $g^{-1}(V)$ is nano open in Y . By hypothesis, $f^{-1}(g^{-1}(V))$ is nanor $^{\wedge}g$ -closed in X . Hence $g \circ f$ is almost nano contra $r^{\wedge}g$ -continuous.

(iii) Let U be a nano regular open set of Z . Since g is almost nano contra g -continuous and Y is $T_{1/2}$ space $g^{-1}(U)$ is nano closed in Y . By hypothesis, $f^{-1}(g^{-1}(U))$ is nanor $^{\wedge}g$ -closed in X . Hence $g \circ f$ is almost nano contra $r^{\wedge}g$ -continuous.

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