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Exploring Fuzzy Sets in Mathematics Science

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Summary: *in mathematics, to get to know Groups fogginess(or Groups not Confirmed)That it is Groups Contains Its elements on grades different from Membership and theory Groups Classic, It is done evaluation membership Elements in group Certain according to system bilateral, where maybe that Belongs to element to The group or no Belongs to To her. In contrast, Allows theory Groups fogginess Evaluation gradually For membership Elements, And that from during function membership take values within period[0,1].It is considered Groups fogginess Generally For groups Classic, where that functions Indicator(known also With functions Featured)For groups Classic Represents Cases especially For functions Membership in Groups fogginess, when take This is amazing functions Values0or1only.in theory Groups fogginess, to get to know Groups same Membership Binary Classic usually In groups clear. maybe application theory Groups fogginess in Fields Multiple where Be Information not Complete or not minute, like Informatics Vitality.*

Keywords: *set, empty set, fuzzy set.*

1. The Introduction :

Fuzzy sets were created in 1965 by the Azerbaijani scientist Lutfi Zadeh from University of California developed it to use as a better way to process data, but his theory Did not receive Interested until 1974.

And There are many motivations that led scientists to develop fuzzy sets. With the development of Computer and Software, the desire arose to invent or program systems that could handle inaccurate information

Fuzzy sets are one of the theories through which such systems can be built. Where memory was restored to many important mathematical works in various fields. mathematics .

On this basis I ate General concepts in regular sets and operations on them. **Then** Fuzzy sets and Types Ha Its properties and the most important algebraic operations on it Also some Fuzzy relationships and Its types.

1.1 The group The Set

Definition (1.1.1): It is any collection of things or a crowd or agglomeration of things that is well and clearly defined, and the things that make up the group are called its elements Or its members.

In general, we symbolize groups with capital letters and their elements with lowercase letters. For example, the set of integers, the set of rational numbers, and the set of real numbers. The sets are expressed in two ways.IQR

List or inventory method:
It is to write the elements of the group inside brackets, with a comma between each element.

$$A = \{1,2,3,4\}$$

Description method:
It is to use a specific description to express the elements of the group, if there is a characteristic or attribute that distinguishes the elements of the group from other elements. There are two methods:
(1) Verbal description:

In this case, we express the group verbally.

$$C = \{X : 10 \text{ أقل من } \quad \text{ عدد زوجي } X\}$$

(2) Symbolic description:
In this case, we express the group in a symbolic way.

$$C = \{X: 1 < X < 5, X \in R\}$$

Definition of the empty set(2.1.1): The empty set It is the group that does not contain any element and is symbolized by the symbol ϕ or { } where $\{\phi\} \neq \phi$ And read Fay.
Definition of a subset(3.1.1): If there are two sets and all the elements of the set are elements of the set such that the set is said to beX, YXYX ≠ YX is a subset of Y and is written XY. If at least one element exists in X and is not in Y, then the set is not a true subset of Y and is written X. $\subseteq \not\subset$ Y.
Definition of power set (Power Set) (4.1.1):
Let a set be a group of all the subsets in it. It is said to be a set of parts and is symbolized by the symbolA AP() (Power set).A
P(A) = We note that from our information, $\{B: B \subseteq A\}$ $\phi \subseteq$ and AA$\subseteq$ AThen we always have ϕ, A$\subseteq$P() whatever the state of the assembly.AA
Definition of belonging and non-belonging:(5.1.1): The symbol means belongs and is used to indicate that the element is in the set.∈ $\notin$ It means does not belong and is used to indicate that the element is not in the set.
Note (1.1.1)

1- The symbol $\notin$ Each of them links an element to a set.∈
2- -2 symbol $\subseteq$, $\not\subset$, $\subset$ Each of them connects one group to another.
3- The empty set is a subset of any set.

Comprehensive groups (The Universal Set) (6.1.1)
identification : It is the group that contains all the elements under study and research, and is often symbolized by the symbol U.
Subset of any set (7.1.1)
identification: For any set, its subsets can be found such that the number of subsets of any set is equal where 2^nn is the number of elements in the set.
Definition of containment Set Inclusion:(8.1.1): Let it be If the following is true, we say that the content of F and E is written as: $(\forall x)(IB \in E \Rightarrow x \in F) \subset F$

Definition of equality:(9.1.1): Let it be F and E are two sets. We say that E equals F if the following equivalence is satisfied: E = F$(\forall x)(x \in E \Leftrightarrow x \in F)$
It is said for both groups E and F are not equal if there is at least one element in one of the sets that does not belong to the other set.
a result (1.1.1):

1- We have E$\subset$E because the implication is always true.$(\forall x)(x \in E \Rightarrow x \in E)$
2- For each group E we have E$\subset \phi$ Because the implication$(\forall x)\left(x \in \phi \Rightarrow x \in E\right)$ Always true.
3- The empty set is unique.
4- Containment is a transitive relationship in the sense that

⊂FF ∩ ⊂G ⇒ E⊂GE

2. Operations on groups: -

Union (Union)
Definition of the Union1.2.1)
Let there be two sets, then the set that consists of all the elements present in any of them is called the union of the two sets and is symbolized by the symbolF. EF. EFE.∪where
{X: x ∈ F ∪ x ∈ E}=FE∪
Definition of intersection (The intersection 2.2.1) :
Let it beF. E Two sets: The set that consists of all the elements found in both of them together is called the intersection of the two sets and is symbolized by the symbol FE.F. EF. E ∩where
{x: x ∈ F ∩ x ∈ E}=FE∩
• If the set is the intersection of the two setsF. E An empty set is said to beF , E separate (Disjoint)
Definition of the difference between groups (The difference between two sets) (2.2.1) :-
Let each of the two sets be called the set whose elements are all the elements that belong toF. EE does not belong to F, but rather to the difference between the two groups F and E, and is symbolized by the symbol FE.−

As follows :FE =− $\left\{x: x \in F \wedge x \notin E\right\}$
And the group EF is defined as:−
EF=− $\left\{x : x \notin F \wedge x \in E\right\}$
Complementary set (Complement of Set
Definition (1.1.13): Let it be F is a subset of the universal set U. The set whose elements are from the universal set that do not belong to F is symbolized by the symbol, i.e. =F^c $\left\{x: x \in U \wedge x \notin F\right\}$ F^c

Note (1.2.1):

1. Let everyone beF, E set if F⊆E Van⊆ E^c F^c
2. Let it beA set of what F =$(F^c)^c$

Analog difference(Analog difference

identification : (5.2.1)

Let each of the two sets be a set, then the symmetric difference of the two sets is the set of the union of the two sets and is symbolized by the symbolF. EF. EFE. (F is read as delta E). That is, (EF)(FE)=FEΔ ∪ Δ

Let everyone beF, E two groups

$F\Delta\phi = \phi$

$F\Delta E = \phi \Leftrightarrow F = E$
We now offer Some properties Operations on groups:
Enjoy The previously defined operations have many properties, some of which we will mention : Let it be A, B, C together We have

$A\cap\phi=\phi$, $A\cup\phi = A$, AA=A -1∩ $A\cup A = A$

-2 Commutative property AB=BA, AB=BA∩∩∪∪

3- Collection feature

$$(A \cap B) \cap C = A \cap (B \cap C)$$

$$(A \cup B) \cup C = A \cup (B \cup C)$$

4- Distributive property

(i) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(ii) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

3. Cartesian product (Cartesian Product)

identification (1.3.1) :

Cartesian product of two non-empty sets E, F represents E F.X

He is the group $\{(a\ , b); a \in E, b \in F\ \}$EF =×

We note that if the group Set A contains n elements and set B contains m elements, then AB has mn elements.××

Now we use some properties of the Cartesian wall.:-

E-1× $\phi = \phi$

2-E FF E, if E × ≠ × ≠ F

3- If the number of elements isE is m and F is n then the number of elements is m ● n The number of EF elements is mn.××

identification : (2.3.1)

Let it be E set how and (where I set evidence) family parts of E.$\{A_i\ , i \in I\}$where Form a group fragmentation E If the following is true:

E = -1$\bigcup_{i \in I} A_i$

2- The parts are intersecting in pairs, which is what we express as:, I j $\neq \phi$ =$A_i \cap A_j$

4. The concept of coverage :

identification (3.3.1) : Let it be E How group and family How groups We say that a family constitutes a cover for a group if the following is true: E$\{B_i\ , i \in I\} \subset \bigcup_{i \in I} B_i$

Fuzzy set

Definition (1.1.2):A fuzzy set is a set of elements consisting of two components, the first component represents the element and the second is the degree of belonging of this element to the subset.

Definition of the degree and function of belonging (membership) (2.1.2)

The degree of belonging of an element determines how close it is to the elements with belonging, and this degree is determined between zero and one. If the degree of belonging of an element is zero, this means that this element is very far from the elements with complete belonging, whose degree of belonging is determined by one. The closer the degree of belonging is to one, the greater the element is among the elements with complete belonging.

The degree of elements of a set can also be measured using a function called the affiliation function.

Definition of the belonging function (3.1.2):
It is a numerical function in the range [0,1] by which the degree of belonging of an element to the fuzzy set is calculated.
Note (1.1.2)

The membership degree of an element to a set can be determined by the affiliation function. $\rightarrow$ I X:A μ Which associates each element with a real number. $\mu_A(x)$

Example (1.1.2)
• If it is, then the element belongs to the group by (0.5) and does not belong to it by (0.5). $\mu_A(x) = 0.5$
• If it is 0.9 and does not belong to it by (0.1). = $\mu_A(x)$

Note (2.1.2)

If we want to know the difference between fuzzy sets and normal sets, we notice that the degree of belonging for a normal set takes only two values, which are 0,1, that is,x :

$$\begin{cases} 1, & x\epsilon A \\ 0, & x \notin A \end{cases} A(x) =$$

Hence, (0,1) ϵ, while if it is a fuzzy set in the setA(x) U For each, the normal set becomes a special case of fuzzy sets., X $0 \leq A(x) \leq 1$x $\in$ X

Definition of the empty fuzzy set(4.1.2) :
We say that a set is empty if and only if the belonging function is equal to zero, i.e.Athat:
$f_A(x) = 0$It is symbolized by the symbol o or ϕ .

Two fuzzy sets are equal

identification(5.1.2):

Let the two fuzzy sets be .A, B We say that they are equal and write if and only if the value of the belonging function is equal to the belonging function.A و BA = BAB

any :

$$f_A(x) = f_B(x)$$

For each. $x \in$ X

We can simply write (where is the element's affiliation function to the group) $f_A = f_B f_B x$ B

Fuzzy group formation

identification: (6.1.2)

Fuzzy sets with the belonging function [0,1] $\rightarrow$:They are called fuzzy sets. Fuzzy sets are written as a set of binaryxμ_ARank(The first component is the element and the second is the degree of belonging)

Any $1 \leq (x)A, \mu x \in X:(()Ax, \mu)\}xA =$

It can be defined by fractions where the numerator represents the degree of belonging and the denominator is the element, i.e. with the phrase {:}. We will symbolize the set whose elements are all the sets whose elements are all the fuzzy sets in the set with the symbol and it is called the fuzzy powers set in. $x \in X \frac{\mu_A}{x} A = xI^x x$

note(3.1.2):

From the previous definition, it appears that the essential difference between normal sets and fuzzy sets is that each element is assigned a degree of belonging. In fact, sometimes a fuzzy set can be determined by the belonging function only, without mentioning the elements independently.

Example: (3.1.2)

Let c be a set defined by the form {cXThe set {(0.2,c),(0.25,b),(0.5,a)} is called a fuzzy set where each element is associated with its degree of belonging and can be written as$X$ = A = X $\} \{ \frac{0.5}{a}, \frac{0.25}{b}, \frac{0.2}{c}$ A =

Example: (4.1.2)

Let {1,2,3,4,5,6,7} be a set. The defined set isX = A

{(7,1){(6,0.7),(5,0.8),(4,0.7),(3,0.5),(2,0.25),(1,0)} is called a fuzzy set in .A = X

Properties of Fuzzy Sets

Fuzzy set conversion (equilibrium) point:

identification(1.2.2) :

Let be a non-empty set, a fuzzy set of which we call a point a transformation point.$XAXx \in X$ For the fuzzy set if 0.5 ()A$A = xf$

Normal fuzzy set

Definition 2.2.2: Let be a non-empty set, a fuzzy set of which we say is a normal fuzzy set.XA XA

1 (): ϵ= $x_0 f_A$ x∃ x_0

That is $\neq \phi$ { 1 : ϵ }= f_A Xx

Height (peak) of the foggy cluster

Definition (2.2.3):

to riseor The vertex of a fuzzy set is symbolized by the symbol () and is defined by the following expression:AAHH() { Sup () : ϵ }$A = f_A xxX$

Especially if they are normal, then 1 $A = H()A$

Fuzzy set holder:

Definition (2.2.4): Let be a fuzzy set in the fuzzy set holder and is denoted by the symbol ().AxAAS orsup p(A)

He knows In the phrase

$sup\, p(A) = \{ \epsilon :) > 0\} x \; X f_A(x)$

Fuzzy group kernel

identification(5.2.2) :

Let be a fuzzy set in , the kernel of the fuzzy set is denoted byAXAker() and you know () / () }AkerA = {x ∈ X f_Ax

Original Fuzzy Set

Definition (2.2.6): Let the fuzzy set be the original set, which we denote by the symbol || and it is known asAXAA

$= \sum_{x \in X}$ f(x) | |A

Example (6.1.2):

Let it beX =[0,1] with β , α And let it be The fuzzy set is known asa, b ∈ RA X In other words:

$$f_A()x = \begin{cases} 0, & \text{اذا} \quad X < a - \alpha \text{أو} \; b + \beta < x \\ 1, & \text{اذا} \quad a < x < b \\ 1 + \left(\frac{x-a}{\alpha}\right), & \text{اذا} \quad a - \alpha < x < a \\ 1 - \left(\frac{b-x}{\beta}\right), & \text{اذا} \quad b < x < b + \beta \end{cases}$$

Then it becomes [0,1],[a] and 1 ().ker(A) = −α, b + β Sup p(A) == HA

Example (7.1.2):

Let it be a blurry set ofx = {1,2,.....6}AxKnowledge of

{, ⟨ 2,0.0 ⟩ , ⟨ 3,0.8 ⟩ , ⟨ 4,1.0 ⟩ , ⟨ 5,0.5 ⟩ , ⟨ 6,1.0 ⟩ ⟩ 1,0.2 ⟨ { {=⟩ (⟨ x , f_A ()xA =

Fuzzy point:

identification (7.2.2):

Let be a non-empty set,X,Y⊆ X The function defined by the formula: $t \in$ I

$$Y_t()x = \begin{cases} t\,; & x \in Y \\ 0\,; & x \notin Y \end{cases}$$

For all to bex ∈ XFuzzy set in.Yx

1- If the set contains only one element, say {, then the function YY = { P Y_t It is written in the form of each

$$x \in X \quad Y_t\,(x) = \begin{cases} t; & x = p \\ 0; & x \neq p \end{cases}$$

It is called a fuzzy point or a fuzzy set.Y

2- If it wast = 1 where t represents the characteristic function Y_t XYX

$$X_{\{P\}} = \begin{cases} 1; & x = p \\ 0; & x \neq p \end{cases}$$

$$\begin{cases} 1; & x \in Y \\ 0; & x \notin Y \end{cases} \quad (x) = X_Y$$

Definition (2.2.8): Let everyone be Two blurry spots in the group.q , p x It is said aboutZ_t, are different (write) if and where and are the two set belonging functions, respectively.Y_t $Y_t \neq Z_t$ $q \neq pZ_tY_t$

Definition (2.2.9): Let be a fuzzy point in the set and a fuzzy set in . $pX\ A\ X$ It is said that it belongs to if it was $(p\ A \leq\ \mu_A(x)$x): where x is the belonging function of the group that contains the element.$Y_t \forall x \in XY_t p$Only.

5. Types of fuzzy sets :

There are different types of fuzzy sets. Classification As follows:

Scattered fuzzy set:

Definition (2.3.1):

A scattering fuzzy set is a discrete fuzzy set whose belonging function is discrete, as in the following example:$X = \{,,\}$: finite set (may be infinite)abc

, $\mu_A \to I$ $\quad A = \left\{\frac{0.3}{a}, \frac{1}{b}, \frac{0.6}{c}\right\}$

continuous fuzzy set

identification (1.2.2) :

Continuous fuzzy set, which is the set whose belonging function is continuous, i.e. $\mu_A : X \to I$ Be continuous.

For example, the function: defined as follows:→ $IR\mu_A$

$$\mu_A\ (x) = \begin{cases} 0.25 & 0 \leq x \leq 4 \\ 0.25(8-x) & 4 \leq\ x\ \leq 8 \\ 0 & x \notin [\,0.8] \end{cases}$$

Complement

Definition (2.3.3): We denote the complement of the fuzzy set by the symbol and it is defined as follows:$A\ \acute{A}$

Complementary properties of the fuzzy set

Let everyone beA, B, CFuzzy clusters in X

1) $A \backslash B = A \cap B^C$
2) $X^C = \emptyset, \emptyset^C = X$
3) $(A^C)^C = A$
4) $(A \cup B)^C = A^C \cap B^C\ , (A \cap B)^C = A^C \cup B^C$
5) If it was$A \leq B$Then$B^C \leq A^C$

note(1.3.2)

C1 andC2 are two essential conditions for the complement to be true.

Example: (1.3.2)

Let the two fuzzy sets be Where :$A_1 A_2$

$$()x\ = \begin{cases} 1 & if\ \ 40 \leq x < 50 \\ 1 - \frac{x-50}{10} & if\ \ 50 \leq x < 60 \quad A_1 \\ 0 & if\ \ 60 < x \leq 100 \end{cases}$$

$$()x = \begin{cases} 0 & if \;\; 40 \le x < 50 \\ \frac{x-50}{10} & if \;\; 50 \le x < 60 \\ 1 - \frac{x-60}{10} & if \;\; 60 \le x < 70 \\ 0 & if \;\; 70 \le x \le 100 \end{cases} \quad A_2$$

ComplementaryA_1she:

$$x) = \begin{cases} 0 & if \; 40 \le x < 50 \\ x - \frac{x-50}{10} & if \;\; 50 \le x < 60 \\ 1 & if \;\; 60 \le x \le 100 \end{cases} \quad) \bar{A}_1$$

Identification Containment (2.3.2):

Let it be AB, two fuzzy sets, we say that a subset of (less than or equal to) is (less than or equal to) if and only if for everyAB AB $f_A \le f_B x \in X$

Definition of union (3.2.2): Let it be A,B are fuzzy sets in , the union of the two sets is the fuzzy set in .XA B X Union of the two groups◌, is the fuzzy setBA

C :$C = A \cup B$

Where its affiliation function is:

$$f_c(x) = Max[f_A, f_B] \,; x \in X$$

In short, you write: $= f_A \cup f_B f_c$

Example: (2.3.2)

Let it be$X$ = {a,b,c}And let everyone be$B$ A, a fuzzy set in , such that$X$

A(a) = 0.2, () = 0.5, () = 0.1, (a) = 0.25, () = 0.125, () = 0.33$AbAcBBbBc$

Van

0.25 = {max { 0.2,0.25 = { max { A(a) , B(a) = (a)(A∪B) -1

max { A(b) , B(b) } = max { 0.5,0125 } = 0.5 = (b)(A)∪B) -2

, B(c) } = max { 0.1,0.33 } = 0.33 = max { A(c) (c)(A∪B) -3

Definition of intersection (4.3.2) :
Let it be Two fuzzy sets in the set , whose belonging function is and , respectively.A, B$X f_A f_B$
Intersection of the two groups It is the fuzzy set such that:A $BCC = A \cap B$

The affiliation function has the following formula:

c= min [,]$f f_A f_B$

Accordingly:(, fc()$xxA \cap B =$

In short, we write:$f_c = f_A \wedge f_B$

Example : If it was

, () = 0.125 , () = 0.33 0.25 = , () 0.1 = , () 0.5 = , () 0.2 = (a)$BbBcBaAcAbA$

Van

, B(a)} = min {0.2, 0.25} = 0.2 min { A(a) = (a)(A⋂B) -1

min {A(b) , B(b)} = min {0.5 , 0.125} = 0.125 = (b)(A)⋂B) -2

min{ A(c) , B(c)} = {min 0.1, 0.33} = 0.1 = (c)() -3$A \cap B$

Algebraic operations on fuzzy sets:

Definition of algebraic sum (2.4.1):

Let it be Two fuzzy sets of the algebraic sum of and denote by the symbol$A$ و $BXAB \oplus B\ A$

$\oplus$ ()() $B = A \cap B \cup \bar{A} \cap BA$

$_A \oplus _B$= max { min[fA (x), 1- fB (x)] , min[1-fA (), fB ()] } fxx

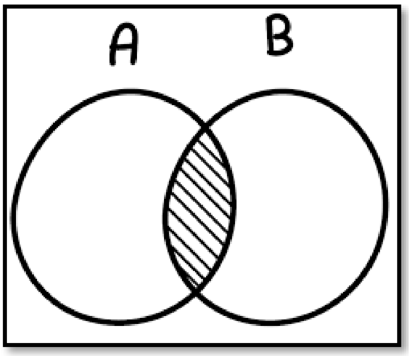

Figure (1) The algebraic sum of two fuzzy sets

Example (1.4.2) :

Let both and be two fuzzy sets.BA:

A = {(, 0.2), (,0.7), (, 1), (, 0)}$x_1x_2x_3x_4$

B = {(, 0.5), (, 0.3), (, 1), (, 0.1)}$x_1x_2x_3x_4$

a result :

A$\oplus$B = ()() ={(,0.5),(,0.7))(,0)(, 0.1)}$A \cap \bar{B} \cup \bar{A} \cap Bx_1x_2x_3x_4$

Fuzzy relationships Fuzzy Relations

Definition: (3.1.1)

Let each of and be a non-empty set. If and are said to be a fuzzy set in, then they are said to be fuzzy sets.XYR ̊$XYRX \times YR \in I^{X \times Y}$

Especially if we say a fuzzy relationship on instead of saying a fuzzy relationship from to, it is also called a fuzzy binary relationship in.$Y = XRXXXX$

• If a fuzzy relation is called a binary relation, we will denote the set whose elements are all fuzzy relations from to by the symbol. In particular, we use the symbol$X = Y\ RX\ XYf\{\mathrm{x.y}\}\ f(x)^2$instead of $f\{\mathrm{x}.x\}$.

• If each of the sets is finite, then it can be represented by a matrix or a diagram. In other words, if YX, R

$$Y = \{y_1, y_2, \dots, y_n\}, X = \{x_1, x_2, \dots, x_n\}$$

The function is defined by the formula $(R: X \times Y \to I\ = r_{ij},).y_i x_i\ R$

So that $j = 1,2, \dots\dots, m$Therefore, it can be represented by a matrix of rank, i.e. it is called a belonging matrix or a fuzzy matrix.$i = 1,2, \dots\dots, n\ Rn \times mR = [r_{ij}]$

Example (3.1.1):

If it is, $Y = \{\, y_1, y_2\} X = \{x_1, x_2, x_3\}$

$$R(x_1, y_1) = 0.7 \quad , \quad R(\,x_1, y_2) = 0.4$$

$$R(x_2, y_1) = 1 \quad , \quad R(x_2, y_2) = 0.2$$

$$R(x_3, y_1) = 0.5 \quad , \quad R(x_3, y_2) = 0.8$$

Van $$R = \begin{bmatrix} 0.7 & 0.4 \\ 1 & 0.2 \\ 0.5 & 0.8 \end{bmatrix}$$

Domain and range domain and range

identification (1.2.3) :

Let be a fuzzy relationship from set to set. It is a fuzzy set in which it is symbolized by the symbol$RXYX$It is known by the formula$dom\ R$

$= max\{R(X, Y): y \in Y\}$ ()($x$dom) for each.$Rx \in X$

Range is a fuzzy set in which the symbol is denoted and known by the formula$RYRanR$

$$y \in Y \text{ لكل } (RanR)(y) = max\{(X, Y): x \in X\}$$

Height is known as the formula R

$$ht\ (R) = max\{\ max\{R(X, Y)\ : y \in Y\}\ : y \in Y\}$$

It is said that it is normal if it is, and the opposite is said that it is partially normal.$R = 1ht(R)$subnormal

It is said that it is a vague self-relationship if it isRX:

$$R(X, Y) = \begin{cases} 1, & x = y \\ 0, & x \neq y \end{cases}$$

It is symbolized by the symbol.$I_{x \times y}$

Inverse Fuzzy Relationships

Definition (3.1.3):

Let be a fuzzy relationship from set to set. The inverse fuzzy relationship is symbolized by $RXYR$,$R^{-1}[r^{-1}{}_{ij}] = R^{-1}$

Its belonging function is defined by the formula:

$\forall x \in X, .\forall y \in Y: \mu_R(x,y) = \mu_R(x,y) = \mu^{-1}{}_R(x,y)$

Theorem (1.3.3) :

If the relationship is blurry on the group, then)$R\ X)^{-1} = RR^{-1}$

Evidence:

Let it be

$$x, y \in X \Rightarrow R(x,y) = R^{-1}(y,x) = (R^{-1})^{-1}(x,y) \Rightarrow (R^{-1})^{-1} = R$$

permission)= $R\ )^{-1}\ R^{-1}$

Example (1.3.3):

$$R^{-1} = \begin{bmatrix} 0.7 & 1 & 0.5 \\ 0.4 & 2 & 0.2 \end{bmatrix}$$

$$R = \begin{bmatrix} 0.7 & 0.4 \\ 1 & 0.2 \\ 0.5 & 0.2 \end{bmatrix}$$

Complex fuzzy relationship

identification (1.4.3):

Let the relationship be blurry from group to group.RX .And a fuzzy relationship from the group to .$YTyZ$

There is a fuzzy relationship from to. Its belonging function is defined by the following expressions:$SX\ Z$

$= max\{min\{R(x,y)\ ,T(y,z)\}: y \in Y\}$ For each () $S(x,z)\ \in X \times Z\ x, z\forall$

That is, the complex fuzzy relationship is called (SComposite Relation) is symbolized by the symbol and it is$R, TR \circ T$

$$\forall(x,z) \in X \times Z \text{ لكل } R^{\circ}\ T(x,z) = max\{min\{R(x,y)\}T(y,z): y \in Y\}$$

Fuzzy equivalence relationship

identification (1.5.3):

Let the relationship be blurred on the group is said aboutRXThat it is$R$

1) Reflective (Reflexive) if 1= (,) and then it is inreflexive if it exists such that (,)$xxR\ \forall x \in X\ \ x \in X \neq 1xxR$

2) Anti-reflective (Antre flexive) if (,) for each.$\neq\ 1xx\ Rx \in X$

3) Symmetrical Symmetric) If each$R(x,y) = R(y,x)x, y \in X$

Therefore, it is asymmetrical. Asymmetric) if found such that.$x, y \in XR(x,y) \neq R(y,x)$

4) Contrasting or anti-symmetrical Ant Symmetric) if the following condition is met:

$$X = Y \text{ فان } R(y,x) > 0\ , R(x,y) > 0$$

5) Transitive (Transitive) if or in other words if$R \circ R \subseteq R$

$$R(x,z) \geq sup_{y \in x}\{min\{R(x,y), R(y,z)\}\}$$

It is said to be a fuzzy equivalence relationship.RFuzzy Equivalence Relations) If the fuzzy relation is reflexive, symmetric, and transitive.R

6. Conclusion :

A fuzzy set is a set of elements consisting of two components, the first component representing the element and the second component representing the degree of belonging of this element to the subset. It is an extension of classical sets in mathematics, as it is used to represent imprecise or ambiguous data.andThe degree of belonging of an element determines how close it is to the elements with belonging, and this degree is determined between zero and one. If the degree of belonging of an element is zero, this means that this element is very far from the elements with complete belonging, whose degree of belonging is determined by one. The closer the degree of belonging is to one, the greater the element is among the elements with complete belonging.

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