INTERNATIONAL JOURNAL FOR INNOVATIVE RESEARCH IN MULTIDISCIPLINARY FIELD
ISSN(O): 2455-0620 [Impact Factor: 9.47]
Monthly, Peer-Reviewed, Refereed, Indexed Journal with IC Value : 86.87
Volume - 11, Issue - 05, May - 2025

DOIs:10.2015/IJIRMF/202505018 --:-- Research Paper / Article / Review

Generalization of Mellin-Stieltjes Transform and its Analytical Structure

[1] **A. N. Rangari**, [2] **R. S. Tathod**, [3] **V. D. Sharma**,
[1] Department of Mathematics, Adarsh College, Dhamangaon Rly.- 444709 (M.S), India.
[2] Department of Mathematics, Shri Shivaji Science College, Amravati, (M.S.), *India.*
[3]Department of Mathematics, Arts, Commerce and Science College, Amravati- 444606(M.S), India.
Email - [1]ashwinarangari@gmail.com, [2]rutujatathod4@gmail.com, [3]vdsharma@hotmail.co.in

Abstract: *Integral Transform is a mathematical operator that transform a function from one space to another space via integration. It is a powerful mathematical tool with a range of applications across various fields. It transforms very complicated differential equation into simpler ones. There are so many integral transforms and each have special contribution in application field. We have reviewed various types of integral transform which are available in the literature and see that Mellin and Stieltjes transform have special efforts to solving difficult problems in science and engineering. So, we have tried to form a new integral transform that is Mellin-Stieltjes transform in the distributional sense. Present paper gives the generalization of Mellin-Stieltjes Transform in the distributional sense. We first define definition of generalized Mellin-Stieltjes Transform, also define some testing function space. The main purpose of this paper is to prove the Analyticity theorem by keeping some parameters fixed.*

Key Words: *Mellin Transform, Stieltjes Transform, Mellin-Stieltjes Transform, Generalized function, Testing function space.*

1. INTRODUCTION:

Integral transforms have a wide range of applications in every field of mathematics, engineering, and physics. There are so many integral transforms in the literature, and each has special importance. Mellin transformation has been applied in many different areas of Physics and Engineering. Maybe the most famous application is the computation of the solution of a potential problem in a wedge-shaped region. They are particularly useful in solving complex differential equations, analyzing asymptotic behaviors, and in the field of signal processing. Understanding Mellin transforms is crucial for engineering students as it provides deeper insight into the behavior of functions and their transformations, which is essential for advanced study and practical applications. [1]. The Mellin transform is named after Finnish mathematician Hjalmar Mellin, who introduced it in 1897. This transform is particularly useful in number theory, mathematical statistics, and the theory of asymptotic expansion. In the study of integral transforms, the Mellin transform has a direct technique for classical boundary and initial value problems. Mellin's name is in honor of R.H. Mellin (1854-1933). Riemann was the first to introduce the Mellin transform, and its explicit formulation was given by Cohen. Mellin transform is used for solving Cauchy’s linear differential equation. [2].

Stieltjes transform is also a very useful integral transform. Stieltjes transform is a generalization of the Riemann integral that allows for integration with respect to a function of bounded variation, finding applications in probability, functional analysis. The Stieltjes integral transform, also known as the Laplace-Stieltjes transform, is named after Thomas Joannes Stieltjes, a Dutch mathematician who made significant contributions to the field of moment problems and continued fractions [3]. Due to the wide range of application of these two integral transforms we combine these transforms with the help of its kernel and form Mellin-stieltjes transform. We will examine all the properties satisfied or not by this newly integral transform. In this paper Mellin-Stieltjes transform is developed in the distributional generalized sense. For the generalization of this integral transform testing function spaces are very mandatory so, we defined testing function spaces in section 3, In section 4, the definition of distributional Mellin-Stieltjes transform is given. In section 5, Analyticity Theorem is proved. Lastly conclusions are given in section 6. Notation and terminologies are as per Zemanian [4] [5].

2. DEFINITION: The conventional Mellin-Stieltjes transform is defined as

$$MS\{f(t,x)\} = F(s,y) = \int_0^\infty \int_0^\infty f(t,x)\, K(t,x)\, dt\, dx$$

where, $K(t,x) = t^{s-1}(x+y)^{-p}$

3. TESTING FUNCTION SPACES:

3.1 The space MS_α :

Let I be the open set in $R_+ \times R_+$ and E_+ denotes the class of infinitely differentiable function defined on I then the space MS_α is given by,

$$MS_\alpha = \{\phi : \phi \in E_+ / \gamma_{b,c,p,l,q}[\phi(t,x)]$$
$$= \sup_{I_1} \left| \lambda_{b,c}(t) t^{q+1} (1+x)^p D_t^l (xD_x)^q \phi(t,x) \right|$$
$$\le C_{lq} A^p p^{p\alpha}\}, \qquad where\ p = 1,2,3,\ldots$$

where the constant A and C_{lq} depends on the testing function $\phi(t,x)$.

$$\lambda_{b,c} = \begin{cases} t^{-b} & 0 < t \le 1 \\ t^{-c} & 1 < t < \infty \end{cases}$$

3.2 The space MS_β :

Let I be the open set in $R_+ \times R_+$ and E_+ denotes the class of infinitely differentiable function defined on I then the space MS_β is given by ,

$$MS_\beta = \{\phi : \phi \in E_+ / \xi_{b,c,p,l,q}[\phi(t,x)]$$
$$= \sup_{I_1} \left| \lambda_{b,c}(t) t^{q+1} (1+x)^p D_t^l (xD_x)^q \phi(t,x) \right|$$
$$\le C_{pq} B^l l^{l\beta}\}, \quad where\ l = 1,2,3,\ldots$$

where the constant B and C_{pq} depends on the testing function $\phi(t,x)$.

3.3 The space MS_α^β :

Let I be the open set in $R_+ \times R_+$ and E_+ denotes the class of infinitely differentiable function defined on I then the space MS_α^β is given by,

$$MS_\alpha^\beta = \{\phi : \phi \in E_+ / \rho_{b,c,p,l,q}[\phi(t,x)]$$
$$= \sup_{I_1} \left| \lambda_{b,c}(t) t^{q+1} (1+x)^p D_t^l (xD_x)^q \phi(t,x) \right|$$
$$\le C A^p p^{p\alpha} B^l l^{l\beta}\}, \ where\ \ p,l = 1,2,3,\ldots$$

where the constant A B and C depends on the testing function $\phi(t,x)$.

4. DISTRIBUTIONAL MELLIN-STIELTJES TRANSFORM :

For $f(t,x) \in MS_\alpha^*$ where MS_α^* is the dual space of MS_α. It contains all distributions of compact support and let $\phi(t,x) \in MS_\alpha$.

Then the distributional Mellin-Stieltjes transform $F(s,y)$ is defined as,

$$MS\{f(t,x)\} = F(s,y) = \left\langle f(t,x)\ ,\ t^{s-1}(x+y)^{-p} \right\rangle, \tag{4.1}$$

where for each fixed t $(0 < t < \infty)$ and x $(0 < x < \infty)$, $s > 0$, $p > 0$ the right hand side of above equation (4.1) has a sense as an application of $f(t,x) \in MS_\alpha^*$ to $t^{s-1}(x+y)^{-p} \in MS_\alpha$.

5. ANALYTICITY OF DISTRIBUTIONAL MELLIN-STIELTJES TRANSFORM:

5.1 Theorem

Let $f(t,x) \in MS_\alpha^*$ and its Mellin-Stieltjes transform $F(s,y)$ is defined as,

INTERNATIONAL JOURNAL FOR INNOVATIVE RESEARCH IN MULTIDISCIPLINARY FIELD
ISSN(O): 2455-0620 [Impact Factor: 9.47]
Monthly, Peer-Reviewed, Refereed, Indexed Journal with IC Value : 86.87
Volume - 11, Issue - 05, May - 2025

$$MS\{f(t,x)\} = F(s,y) = \left\langle f(t,x)\ ,\ t^{s-1}(x+y)^{-p} \right\rangle$$

then $F(s,y)$ is analytic for some fixed $s>0$ and $y>0$ and

i) $D_s F(s,y) = \left\langle f(t,x)\ ,\ \frac{\partial}{\partial s} t^{s-1}(x+y)^{-p} \right\rangle$

ii) $D_y F(s,y) = \left\langle f(t,x)\ ,\ \frac{\partial}{\partial y} t^{s-1}(x+y)^{-p} \right\rangle$

Proof: *i)* Let s and y be an arbitrary but fixed. Choose the real positive numbers a_1 , b_1 and r such that $\sigma_1 < a_1 < \operatorname{Re} s - r < \operatorname{Re} s + r < b_1 < \sigma_2$.

Also let Δs be a complex increment such that, $0 < |\Delta s| < r$

For $\Delta s \neq 0$

Consider,

$$\frac{F(s+\Delta s,y)-F(s,y)}{\Delta s} - \left\langle f(t,x), \frac{\partial}{\partial s} t^{s-1}(x+y)^{-p} \right\rangle$$

$$= \frac{1}{\Delta s}\int_0^\infty\int_0^\infty f(t,x)t^{(s+\Delta s)-1}(x+y)^{-p}\,dt\,dx - \frac{1}{\Delta s}\int_0^\infty\int_0^\infty f(t,x)t^{s-1}(x+y)^{-p}\,dt\,dx - \left\langle f(t,x), \frac{\partial}{\partial s} t^{s-1}(x+y)^{-p} \right\rangle$$

$$= \frac{1}{\Delta s}\int_0^\infty\int_0^\infty f(t,x)[t^{(s+\Delta s)-1} - t^{s-1}](x+y)^{-p}\,dt\,dx - \left\langle f(t,x), \frac{\partial}{\partial s} t^{s-1}(x+y)^{-p} \right\rangle$$

$$= \left\langle f(t,x)\ ,\ [t^{(s+\Delta s)-1} - t^{s-1}]\frac{(x+y)^{-p}}{\Delta s} \right\rangle - \left\langle f(t,x), \frac{\partial}{\partial s} t^{s-1}(x+y)^{-p} \right\rangle$$

$$= \left\langle f(t,x)\ ,\ [t^{(s+\Delta s)-1} - t^{s-1}]\frac{(x+y)^{-p}}{\Delta s} - \frac{\partial}{\partial s} t^{s-1}(x+y)^{-p} \right\rangle$$

$$= \langle f(t,x)\ ,\ \psi_{\Delta s} \rangle$$

where, $\psi_{\Delta s} = [t^{(s+\Delta s)-1} - t^{s-1}]\frac{(x+y)^{-p}}{\Delta s} - \frac{\partial}{\partial s} t^{s-1}(x+y)^{-p}$

To prove $\psi_{\Delta s}(t,x) \in MS_\alpha$ we shall show that as $|\Delta s| \to 0$, $\psi_{\Delta s}(t,x)$ converges to zero in MS_α .

Let C denotes the circle with center at s and radius r_1 where $0 < r < r_1 < \min(s-a_1, b_1 - s)$.

We may interchange differentiation on s with differentiation on t and by using Cauchy's integral formula.

$$(D_t)^l \psi_{\Delta s}(t,x) = \frac{(x+y)^{-p}}{\Delta s}[P(s+\Delta s-l)t^{(s+\Delta s)-l-1} - P(s-l)t^{s-l-1}] - \frac{\partial}{\partial s}P(s-l)t^{s-l-1}(x+y)^{-p}$$

Where $P(s+\Delta s-l)$ is polynomial in $(s+\Delta s-l)$ and $P(s-l)$ is polynomial in $(s-l)$

Now by applying Cauchy's integral formula,

$(D_t)^l \psi_{\Delta s}(t,x)$

$$= \frac{(x+y)^{-p}}{\Delta s}\left\{\frac{1}{2\pi i}\int_C \frac{P(z-l)t^{z-l-1}}{z-(s+\Delta s)}dz - \frac{1}{2\pi i}\int_C \frac{P(z-l)t^{z-l-1}}{z-s}dz\right\} - \frac{1}{2\pi i}\int_C \frac{P(z-l)t^{z-l-1}}{(z-s)^2}(x+y)^{-p}\,dz\Bigg\}$$

$$= \frac{(x+y)^{-p}}{2\pi i\,\Delta s}\int_C \left[\frac{1}{z-(s+\Delta s)} - \frac{1}{z-s}\right]P(z-l)t^{z-l-1}\,dz - \frac{(x+y)^{-p}}{2\pi i}\int_C \frac{P(z-l)t^{z-l-1}}{(z-s)^2}dz$$

$$=\frac{(x+y)^{-p}}{2\pi i\,\Delta s}\int_C\left[\frac{\Delta s}{(z-s-\Delta s)(z-s)}\right]P(z-l)t^{z-l-1}\,dz-\frac{(x+y)^{-p}}{2\pi i}\int_C\frac{P(z-l)t^{z-l-1}}{(z-s)^2}\,dz$$

$$=\frac{(x+y)^{-p}}{2\pi i}\int_C\left[\frac{1}{(z-s-\Delta s)(z-s)}-\frac{1}{(z-s)^2}\right]P(z-l)t^{z-l-1}\,dz$$

$$=\frac{(x+y)^{-p}}{2\pi i}\int_C\frac{\Delta s}{(z-s-\Delta s)(z-s)^2}\,P(z-l)t^{z-l-1}\,dz$$

$$=\frac{(x+y)^{-p}\,\Delta s}{2\pi i}\int_C\frac{1}{(z-s-\Delta s)(z-s)^2}\,P(z-l)t^{z-l-1}\,dz$$

Now,

$$D_t^l(xD_x)^q\,\psi_{\Delta s}(t,x)\;=\frac{\Delta s}{2\pi i}P(-p-q)(x+y)^{-p-q}x^q\int_C\frac{P(z-l)t^{z-l-1}}{(z-s-\Delta s)(z-s)^2}\,dz$$

$$=\frac{\Delta s\,P(-p-q)(x+y)^{-p-q}x^q}{2\pi i}\int_C\frac{P(z-l)t^{z-l-1}}{(z-s-\Delta s)(z-s)^2}\,dz$$

Now for all $z\in C$ and $0<t<\infty$, $\sup_{I_1}\left|\lambda_{b,c}(t)t^{q+1}(1+x)^pP(-p-q)(x+y)^{-p-q}x^q\right|\le K$

8where K is constant independent of z and t.

Moreover, $|z-s-\Delta s|>r_1-r>0$ and $|z-s|=r_1$,

$C_1=max\left\{\left|P(z-l)t^{z-l-1}\right|:z\in C\right\}$

Consequently,

$$\sup_{I_1}\left|\lambda_{b,c}(t)t^{q+1}(1+x)^p\,D_t^l(xD_x)^q\,\psi_{\Delta s}(t,x)\right|$$

$$=\sup_{I_1}\left|\lambda_{b,c}(t)t^{q+1}(1+x)^p\,\frac{\Delta s\,P(-p-q)(x+y)^{-p-q}x^q}{2\pi i}\int_C\frac{P(z-l)t^{z-l-1}}{(z-s-\Delta s)(z-s)^2}\,dz\right|$$

$$\le\sup_{I_1}\left|\lambda_{b,c}(t)t^{q+1}(1+x)^pP(-p-q)(x+y)^{-p-q}x^q\right|\frac{|\Delta s|}{2\pi}\int_C\frac{\left|P(z-l)t^{z-l-1}\right|}{|z-s-\Delta s||z-s|^2}|dz|$$

$$\le K\frac{|\Delta s|}{2\pi}\int_C\frac{C_1}{(r_1-r)(r_1)^2}|dz|$$

$$\le\frac{|\Delta s|}{2\pi}\int_C\frac{KC_1}{(r_1-r)(r_1)^2}|dz|$$

$$\le\frac{|\Delta s|}{2\pi}\int_C\frac{C_2}{(r_1-r)(r_1)^2}|dz|$$

Where $C_2=KC_1$

$$\leq \frac{|\Delta s|}{2\pi} \frac{C_2}{(r_1 - r)(r_1)^2} 2\pi r_1$$

$$\leq |\Delta s| \frac{C_2}{(r_1 - r)(r_1)}$$

The right-hand side is independent of t and converges to zero as $|\Delta s| \to 0$

This shows that $\psi_{\Delta s}(t,x)$ converges to zero in MS_α as $|\Delta s| \to 0$

ii) Let s and y be an arbitrary but fixed. Choose the real positive number a_2 , b_2 and h such that $\sigma_1' < a_2 < \operatorname{Re} y - h < \operatorname{Re} y + h < b_2 < \sigma_2'$.

Also let Δy be a complex increment such that, $0 < |\Delta y| < h$

For $\Delta y \neq 0$

We write,

$$\frac{F(s, y+\Delta y) - F(s,y)}{\Delta y} - \left\langle f(t,x), \frac{\partial}{\partial y} t^{s-1}(x+y)^{-p} \right\rangle$$

$$= \frac{1}{\Delta y}\int_0^\infty\int_0^\infty f(t,x) t^{s-1}(x+y+\Delta y)^{-p}\, dt\, dx - \frac{1}{\Delta y}\int_0^\infty\int_0^\infty f(t,x) t^{s-1}(x+y)^{-p}\, dt\, dx - \left\langle f(t,x), \frac{\partial}{\partial y} t^{s-1}(x+y)^{-p} \right\rangle$$

$$= \frac{1}{\Delta y}\int_0^\infty\int_0^\infty f(t,x)[(x+y+\Delta y)^{-p} - (x+y)^{-p}] t^{s-1}\, dt\, dx - \left\langle f(t,x), \frac{\partial}{\partial y} t^{s-1}(x+y)^{-p} \right\rangle$$

$$= \left\langle f(t,x), [(x+y+\Delta y)^{-p} - (x+y)^{-p}] \frac{t^{s-1}}{\Delta y} \right\rangle - \left\langle f(t,x), \frac{\partial}{\partial y} t^{s-1}(x+y)^{-p} \right\rangle$$

$$= \left\langle f(t,x), [(x+y+\Delta y)^{-p} - (x+y)^{-p}] \frac{t^{s-1}}{\Delta y} - \frac{\partial}{\partial y} t^{s-1}(x+y)^{-p} \right\rangle$$

$$= \left\langle f(t,x), \psi_{\Delta y}(t,x) \right\rangle$$

where, $\psi_{\Delta y}(t,x) = [(x+y+\Delta y)^{-p} - (x+y)^{-p}] \dfrac{t^{s-1}}{\Delta y} - \dfrac{\partial}{\partial y} t^{s-1}(x+y)^{-p}$

To prove $\psi_{\Delta y}(t,x) \in MS_\alpha$ we shall show that as $|\Delta y| \to 0$, $\psi_{\Delta y}(t,x)$ converges to zero in MS_α.

To proceed, let C_1 denotes the circle with centre at y and radius h_1,
where $0 < h < h_1 < min(y - a_2, b_2 - y)$.
We may interchange differentiation on y with differentiation on x and by using Cauchy's integral formula.

$$(xD_x)^q \psi_{\Delta y}(t,x) = x^q \frac{t^{s-1}}{\Delta y}[P(-p-q)(x+y+\Delta y)^{-p-q} - P(-p-q)(x+y)^{-p-q}]$$
$$- x^q \frac{\partial}{\partial y} t^{s-1} P(-p-q)(x+y)^{-p-q}$$

Where $P(-p-q)$ is polynomial in $(-p-q)$.

Now by applying Cauchy's integral formula,

$(xD_x)^q \psi_{\Delta y}(t,x)$

$$= x^q \frac{t^{s-1}}{\Delta y}\left\{\frac{1}{2\pi i}\int_C \frac{P(-p-q)(x+z)^{-p-q}}{z-(y+\Delta y)}dz - \frac{1}{2\pi i}\int_C \frac{P(-p-q)(x+z)^{-p-q}}{z-y}dz\right\} - x^q \frac{t^{s-1}}{2\pi i}\int_C \frac{P(-p-q)(x+z)^{-p-q}}{(z-y)^2}dz$$

$$= \frac{x^q t^{s-1}}{2\pi i\,\Delta y}\int_C \left[\frac{1}{z-(y+\Delta y)} - \frac{1}{z-y}\right] P(-p-q)(x+z)^{-p-q}\,dz - \frac{x^q t^{s-1}}{2\pi i}\int_C \frac{P(-p-q)(x+z)^{-p-q}}{(z-y)^2}dz$$

$$= \frac{x^q t^{s-1}}{2\pi i\,\Delta y}\int_C \left[\frac{\Delta y}{(z-y-\Delta y)(z-y)}\right] P(-p-q)(x+z)^{-p-q}\,dz - \frac{x^q t^{s-1}}{2\pi i}\int_C \frac{P(-p-q)(x+z)^{-p-q}}{(z-y)^2}dz$$

$$= \frac{x^q t^{s-1}}{2\pi i}\int_C \left[\frac{1}{(z-y-\Delta y)(z-y)} - \frac{1}{(z-y)^2}\right] P(-p-q)(x+z)^{-p-q}\,dz$$

$$= \frac{x^q t^{s-1}}{2\pi i}\int_C \frac{\Delta y}{(z-y-\Delta y)(z-y)^2} P(-p-q)(x+z)^{-p-q}\,dz$$

$$= \frac{x^q t^{s-1}\Delta y}{2\pi i}\int_C \frac{1}{(z-y-\Delta y)(z-y)^2} P(-p-q)(x+z)^{-p-q}\,dz$$

Now,

$$D_t^l (xD_x)^q \psi_{\Delta y}(t,x) = \frac{\Delta y}{2\pi i} P(s-l) t^{s-l-1} x^q \int_C \frac{P(-p-q)(x+z)^{-p-q}}{(z-y-\Delta y)(z-y)^2}dz$$

Now for all $z \in C_1$ and $0 < x < \infty$, $\sup_{I_1}\left|\lambda_{b,c}(t)t^{q+1}(1+x)^p x^q P(s-l)t^{s-l-1}\right| \le K$

where K is constant independent of z.

Moreover, $|z-s-\Delta s| > h_1 - h > 0$ and $|z-s| = h_1$

$C_1 = max\left\{\left|P(-p-q)(x+z)^{-p-q}\right| : z \in C_1\right\}$

Consequently,

$$\sup_{I_1}\left|\lambda_{b,c}(t)t^{q+1}(1+x)^p D_t^l (xD_x)^q \psi_{\Delta y}(t,x)\right|$$

$$= \sup_{I_1}\left|\lambda_{b,c}(t)t^{q+1}(1+x)^p \frac{\Delta y\, x^q P(s-l)\, t^{s-l-1}}{2\pi i}\int_C \frac{P(-p-q)(x+z)^{-p-q}}{(z-y-\Delta y)(z-y)^2}dz\right|$$

$$\le \sup_{I_1}\left|\lambda_{b,c}(t)t^{q+1}(1+x)^p x^q P(s-l)\, t^{s-l-1}\right| \frac{|\Delta y|}{2\pi}\int_C \frac{\left|P(-p-q)(x+z)^{-p-q}\right|}{|z-y-\Delta y||z-y|^2}|dz|$$

$$\le K\frac{|\Delta y|}{2\pi}\int_C \frac{C_1}{(h_1-h)(h_1)^2}|dz|$$

$$\le \frac{|\Delta y|}{2\pi}\int_C \frac{KC_1}{(h_1-h)(h_1)^2}|dz|$$

$$\le \frac{|\Delta y|}{2\pi} \int_C \frac{C_2}{(h_1 - h)(h_1)^2} |dz|$$

Where $C_2 = KC_1$

$$\le \frac{|\Delta y|}{2\pi} \frac{C_2}{(h_1 - h)(h_1)^2} 2\pi h_1$$

$$\le |\Delta y| \frac{C_2}{(h_1 - h)(h_1)}$$

The right-hand side is independent of x and converges to zero as $|\Delta y| \to 0$

This shows that $\psi_{\Delta y}(t, x)$ converges to zero in MS_α as $|\Delta y| \to 0$

Which ends the proof.

6. CONCLUSION:

In this paper we developed a new integral transform that is Mellin-Stieltjes transform which is generalized in the distribution sense some testing functions are defined which are helpful to prove the analyticity theorem. The main aim of this paper is to prove the analyticity theorem and prove that, Mellin-Stieltjes transform is also analytic which will help to solve various types of differential equation.

REFERENCES:

1. Ansari Alireza, (2012): Generalized Product Theorem for the Mellin Transform And its Applications, *Kragujevac Journal of Mathematics*, Vol. 36 No. 2 pp.287-298.
2. John Graham le Maistre Pratt, (July 2000) : *Application of the Fourier-Mellin transform to translation-, rotation- and scale-invariant plant leaf identification Thesis*, Mc Grill University, Montreal.
3. Pathak Ram Sankar, Debnath Lokenath, (1987) : Recent developments on the Stieltjes transform of generalized functions, *International Journal of Mathematics and Mathematical Sciences, Hindawi*, Vol. 10, pages 1-30.
4. Zemanian, A. H., (1968). *Generalized integral transform, Inter science publisher* New York.
5. Zemanian, A.H., (1965). *Distribution Theory and Transform Analysis* McGraw Hill, New York.
6. Debnath Lokenath and Bhatta Dambaru, (2007). *Integral Transforms and their Applications* Chapman and Hall/CRC Taylor and Francis Group Boca Raton London, New York.
7. Pathak R. S., *Integral transformation of generalized functions and their applications* Hordon and Breach science Publishers, Netherland.
8. Gelfand, I. M. and Shilov, G.E., (1968). *Generalized function* Vol. II, Academic Press, New York.
9. Sharma, V. D. and Rangari, A. N., (2015): Fourier-Laplace Transforms of Some Special Functions, *International Journal of Engineering Research and General Science*, Volume 3, Issue 6, pp. 922-928.
10. Rangari, A. N. and Sharma, V. D., (2021): Study of 2D Fourier-Finite Mellin Transform and Its Inversion, *International Journal of Recent Scientific Research*, Volume 12, Issue 3, pp. 41297-41306.
11. Sharma, V.D. and Dolas, P. D., (2012) : Analyticity of Distribution Generalized Fourier-Stieltjes Transforms, *Int. Journal of Math. Analysis*, Vol. 6, No. 9, pp. 447-451.
12. Sharma, V. D. and Rangari, A. N., (2011) : Analyticity of Distributional Fourier-Laplace Transform, *International Journal of Mathematical Sciences and Engineering Applications*, Vol. 5. No. V, pp. 57-62.
13. Deshmukh, P. R. and Gudadhe, A. S., (2012): Analyticity Theorem for Fractional Laplace Transform, *Scientific. Reviews and Chemical Communications*, 2(3), pp. 277-281.