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Comparative analysis of Block Maxima method and Peak over Threshold method using Value at Risk forecasts of the South African Industrial Index

[1] Owen Jakata and [2] Peter Chimwanda
[1] Lecturer, Department of Human Resources Management, Faculty of Commerce, Bindura University of Science Education, Bindura, Zimbabwe
[2] Lecturer, Department of Mathematics and Statistics, School of Natural Sciences and Mathematics, Chinhoyi University of Technology, Chinhoyi, Zimbabwe.

Email - [1]owenjakata@rocketmail.com and [2]pchimwanda@gmail.com

Abstract: *This study makes a comparative analysis of the Extreme Value Theory (EVT) methods using the monthly South African Industrial Index (J520) returns. The purpose is to investigate the comparative performance of the Block Maxima (BM) and Peak over Threshold (PoT) methods in the estimation of Value at Risk (VaR) for the period 1995 to 2018. The monthly return distribution is fitted to the Generalised Extreme Value Distribution (GEVD) and Generalised Pareto Distribution (GPD) using the maxima/minima extracted from the BM and PoT methods respectively. Parameters are estimated using the Maximum Likelihood Method (MLE) method. The parameter estimations show that the PoT method is the preferable model as it has a superior goodness of fit performance. The findings reveal that the VaR forecasts of the BM method are less than the PoT method. The results also reveal that for both methods the probability of losses is less than the probability of gains. Backtesting was carried out using the Kupiec test which validates that the PoT method is more preferable than the BM method. The contribution of this study is that it supports the evidence that the PoT method VaR forecasts are more reliable and satisfactory than the BM method VaR forecasts.*

Key Words: *Back-testing; Block Maxima method; Extreme value theory; Generalized Extreme Value Distribution; Generalized Pareto Distribution; Kupiec Test; monthly South African Industrial Index (J520) returns; Peaks over Threshold method; Value at Risk.*

Highlight of this study

- the parameter estimations show that the PoT method is the preferable method as it has a better goodness of fit performance.
- the VaR forecasts for BM method are less than the VaR forecasts for the PoT method.
- the PoT method produces more reliable and satisfactory VaR forecasts than the BM method as validated by back-testing

The study contributes to the evidence that supports that the PoT method performs better than the BM method

1. INTRODUCTION

This study explores the comparative analysis of Extreme Value Theory (EVT) methods in modelling extreme events in financial markets using the Block Maxima (BM) method and Peak-over-Threshold (PoT) method. The EVT is the methodology used for the assessment of the investors, researchers and practitioners' rate of exposure to risk which is termed the Value at Risk (VaR). The BM and PoT methods are the main methods used to estimate VaR. McNeil (1999) states that VaR is the maximum potential loss in value of a financial asset for a given probability level over a period of time. Alternatively, Value at Risk estimates the maximum potential loss in value which an institution is exposed to in the stock market. According to Halilbegovic and Vehabovic (2012), VaR is the main methodology for managing risk and is applied in combination with other techniques to obtain optimal business results thereby maximising shareholder's wealth.

EVT is a robust methodology which is increasingly used in the estimation of VaR. The EVT models are grouped into two: the unconditional (static) and conditional (they use the GARCH method to model the heteroscedasticity) (Abad et al., 2013). The heteroscedasticity of the monthly returns in this study is insignificant hence the adoption of the static models. This study compares the unconditional models based on the univariate EVT which are divided into two methods used in modelling a return distribution:

i) the Block Maxima (BM) method,
ii) the Peaks over Threshold (PoT) method.

Szubzda and Chlebus (2019) states that there is no unambiguous answer which one between the BM and PoT method is more effective.

The Generalised Pareto Distribution (GPD) method is a two-parameter model which uses the PoT method to extract excesses above a certain threshold and has proved to be one of the best ways to apply EVT in practice. The main problem is that of choosing the optimal threshold. According to Gilli and Kellezi (2006), the GPD method is better in modelling insufficient data than the three parameter Generalised Extreme Value Distribution (GEVD) method which requires the use of large datasets. Flugentiusson (2012) states that BM method is inferior to the POT method when using a financial time series. The PoT method is considered to perform better than the BM method in some instances.

Allen, Singh and Powell (2011), stated that the BM method avoids the dependency problem in the dataset which tends to complicate the use of the threshold method when applying the PoT method. The PoT method is one of the most widely used modelling methods for fitting distributions above a sufficiently high threshold (Sigauke, Makhwiting and Lesaoana, 2014). The estimates of extreme events provided by the BM method may underestimate the extreme events in some cases (Makhwiting, Sigauke and Lesaoana, 2014). Studies by Bali (2007) and Tolikas (2011) show that the BM method is a good approach to financial risk estimation. However, Bucher and Zhou (2018), stated that the PoT method is preferable for quantile estimation of VaR while BM method is preferable for return level estimation. According to Cerovic and Karadzic (2012) the PoT method estimates are more accurate and consistent than the BM method.

Heymans and Santana (2017) confirmed that some sub-indices such as the South African Industrial Index (J520) are not always as information efficient as the South Africa's Johannesburg Stock Exchange (JSE) All Share Index (ALSI) thereby allowing investors the possibility of making excess profits/losses (extreme gains/losses). The sub-indices of the ALSI are therefore modelled by Extreme Value Distributions (EVD) such as the GEVD and GPD.

This study investigates the application of the two EVT methods that estimate VaR using the monthly South African Industrial Index (J520) returns. The Kupiec test is used for back-testing by evaluating the BM and PoT methods VaR forecasts for reliability and accuracy. The EVT method from McNeil, Frey & Embrechts (2005) is adopted in this study.

1.1 Statement of the Problem

In the last three decades, Global stock markets, which include the South African stock market have been exposed to significant instabilities caused by various international financial crises which include: the Asia-Pacific Financial Crisis, the Global Financial Crisis and the Chinese Stock Market Crash. This resulted in the criticism of the traditional risk models and it motivated researchers to build models that predict rare events that have caused financial market disasters. Many concepts have been developed for managing extreme risk as there is no one method that can accurately predict the extreme risk in advance. In this study backtesting is therefore used to evaluate BM and PoT methods VaR forecasts for reliability and accuracy using the monthly South African Industrial Index (J520) returns.

1.2 Justification of the Study

This study investigates the reliability and accuracy of the BM and PoT methods which is important for managing extreme risk. The VaR forecast is a widely used metric in the financial market. Since there is no method which predicts accurate VaR forecasts, back-testing should be undertaken. Back-testing is carried out to confirm the reliability and accuracy of the VaR model validation.

1.3 Objectives of the study

This study investigates the comparative performance of the BM and the PoT methods in terms of the VaR forecasts.

The specific objectives of the study are:

- To investigate the application of the BM and the PoT methods by fitting the monthly returns of the South African Industrial Index (J520).

- To estimate BM and the PoT methods VaR forecasts and make a comparative analysis.

This study contributes to the building of the BM and PoT methods used in the estimation of VaR used for the comparative analysis. It differs from other studies in that it uses in the monthly South African Industrial Index (J520) to model the BM and the PoT methods VaR forecasts. This study is organized as follows. Section 2: Literature review, Section 3 Methodology and data, Section 4: Results and Discussion, Section 5 Conclusions are presented.

2. Literature Review

EVT is a well-developed method that is used to modelling the financial impact of extreme events using the two tails of a return distribution. Many of the research studies have been carried out in this area which include: Hakim (2018) Omari, Mwita and Waititu (2017), Nortey, Asare and Mettle (2016), Gilli and Këllezi (2006), Embrechts. Klüppelberg and Mikosch (2012). Several researchers have discussed the comparative performance of BM and PoT method, which include the following studies discussed in this section.

Cerovic and Karadzic (2015) investigated the comparative performance of the BM and PoT methods using the EVT in the Montenegrin equity market. Their study investigated whether the PoT method calculates VaR more accurately and consistently than the BM method. The researchers analysed the MONEX20 index daily returns of the Montenegrin equity market for the period 2004 – 2014. The Kupiec test revealed that the PoT method estimates the VaR more accurately and consistently than the BM method.

Bucher and Zhou (2018), investigated the performance of two EVT methods: the BM method and the PoT method. The researchers stated that the PoT method is better than the BM method at utilising extreme observations more efficiently. They argued that the method you choose depends on the ultimate statistical interest: PoT is more suitable for quantile estimation of VaR and ES, while BM is more suitable for return level estimation.

Gilli and Kellezi (2006) used both the BM and PoT method for modelling extreme risk: VaR, Expected Shortfall (ES) and Return level. The researchers concluded that the PoT method is better as it utilises the return distribution more efficiently.

EVT is applied in this study as in Ngailo (2016) and Lazoglou & Anagnostopoulou (2017) and Schmidt F, Zhou X and Toutlemonde F (2014) who used the GEVD and GPD to make a comparative analysis of the BM and the PoT method. This study differs with these studies as it uses the South African Industrial Index (J520).

3. Methodology and data

3.1 Extreme Value Theory

According to Ender and Ma (2014), the first insights in EVT were published by Fisher and Tippett in 1928. Significant contributions to the statistical modelling of extremes were followed by Jenkinson 1955 for GEVD and Pickands 1975 for GPD. EVT is used to build models for analysing the tail distribution of a financial return distribution which was proposed by Fisher and Tippett (1928) theorem. In this study, the return series is fitted to the BM and PoT methods and VaR is estimated, a comparative analysis of the two methods is undertaken.

3.2 The Generalized Extreme Value Distribution

The BM method is the most traditional method of modelling extreme events and is used to extract maxima/minima for the return distribution. The disadvantage of this method is that it is considered wasteful as it does not utilize all the extreme values in the same block. In practice PoT method is considered more efficient as it is increasingly being used as all the data exceeding a certain threshold is used.

The BM method is considered when the maxima/minima value of each block are extracted from independent and identically distributed variables (McNeil, 1999) which converges to the GEVD. The asymptotic distribution of the maximum observations is represented by three distributions which are Fretchet, Weibull and Gumbel (Fisher and Tippett, 1928). The limit distribution function of the three distributions converges to the GEVD model (Jenkinson, 1955) which is defined as:

$$F_{\xi,u,\sigma}(x) = \exp\left(-\left[1 + \xi\left(\frac{x-u}{\sigma}\right)\right]^{\frac{-1}{\xi}}\right) \quad \text{if } \xi \neq 0 \qquad (1)$$

$$F_{\xi,u,\sigma}(x) = \exp\left[-\exp\left(\left(-\frac{x-u}{\sigma}\right)\right)\right] \quad \text{if } \xi \to 0 \qquad (2)$$

where $\sigma > 0$ and $1 + \xi\left(\frac{x-u}{\sigma}\right) > 0$, μ is defined as location parameter, σ is defined as a scale parameter and ξ is defined as the shape parameter. When $\xi > 0$, F belongs to the fat-tailed Frechet class distribution which is appropriate for fat-tailed financial data. When $\xi < 0$, F will belong to the short-tailed negative Weibull class distribution. And, as $\xi \rightarrow 0$, F tends to the light-tailed Gumbel class distribution. Practically the return distribution is divided into blocks and the maxima/minima is extracted. The parameters are estimated using the maximum likelihood estimation (MLE) (Coles, 2001).

3.2.1 Estimation of the Value-at-risk for the Generalised Extreme Value Distribution

The VaR of a return distribution for limiting GEVD (Cerovic and Karadzic, 2006) is given by

$$\widehat{VaR_p} = u - \frac{\hat{\sigma}}{\hat{\xi}}\left(1-(-n\ln(1-p))^{-\hat{\xi}}\right) \quad (3)$$

Where n is the block size.

3.3 The Generalised Pareto Distribution

The second method used to analyse the return distribution of extreme events is called PoT method which extracts values that exceed a certain threshold and converge to a GPD when the threshold is sufficiently high (Balkema and de Haan, 1974 and Pickands, 1975) The limiting distribution function for GPD is given by:

$$G_{\xi,\beta}(x) = \begin{cases} 1-\left(1+\frac{\xi x}{\beta}\right)^{-1/\xi} & \text{if } \xi \neq 0 \\ 1-e^{-x/\beta} & \text{if } \xi = 0 \end{cases} \quad (4)$$

Where x>0 when $\xi \geq 0$, $0 \leq x \leq -\beta/\xi$ when $\xi < 0$ and β>0 with β is the scale parameter and ξ is the shape parameter.

The parameters are estimated using the method of probability weighted moments, the L-moments, or with the MLE (Hosking et al., 1984). The MLE method is used to estimate the parameters in this study. Choosing the optimal threshold can be a problem. The threshold should be sufficiently high so that the distribution converges to a GPD. The Pareto QQ plot is the method of optimal threshold selection which is used in this study as in Sigauke, Makhwiting and Lesaoana (2014).

3.3.1 Estimation of the Value at Risk for the Generalised Pareto Distribution

The VaR for a given probability p can be defined as the p-th quantile of F.

$$VaR_p = F^{-1}(1-p) \quad (5)$$

Where F^{-1} is the quantile function, for the GPD:

$$\widehat{VaR_p} = \begin{cases} u + \frac{\hat{\beta}}{\hat{\xi}}\left(\left(\frac{n}{N_u}\right)^{-\hat{\xi}} - 1\right) & \xi \neq 0 \\ u - \hat{\beta}\ln\left(\frac{n}{N_u}(1-p)\right) & \xi = 0 \end{cases} \quad (6)$$

where $\hat{\xi}$ and $\widehat{\beta}$ are maximum likelihood estimates of the GPD estimates.

VaR is not a coherent risk estimate. Coherence means that the risk estimates will still behave sensibly with loss distributions being subject to alteration or combination. The PoT method is much more stable than the BM method since VaR estimation is very sensitive to changes in the size of blocks.

3.4.0 Evaluating the performance of the BM and PoT methods using VaR forecasts.

Back-testing is a statistical method used to compare actual profits and losses with corresponding VaR forecasts (Kupiec, 1995). Back-testing is used to validate whether the obtained VaR forecasts for each model are reliable and satisfactory (Halilbegovic and Vehabovic, 2016). In this study the Kupiec Test which is an unconditional coverage test is used to determine the number of VaR exceedances above the actual losses/gains.

Kupiec (POF) Test

The Kupiec Test is also known as the likelihood ratio test. In this test N is the observed number of exceedances in the sample when the loss/gain is larger than the VaR estimate and T is the total number of observations. The Kupiec

POF test (proportion of failure) checks whether the number of exceedances is in accordance with the level of confidence. The null hypothesis for the POF test is expressed as:

H_0:p = $\frac{N}{T}$
T = is the total number of observations
N= number of exceedances in the sample
p = proportion of failure

Kupiec (1995), stated that the proportion of failure (POF) test is best applied as a likelihood ratio test. Halilbegovic and Vehabovic (2016) stated that the likelihood ratio test statistic formula is:

$$\text{Kupiec POF} = -2ln\,\frac{(1-p)^{T-N} \;*\; P^{N}}{(1-\frac{N}{T})^{T-N}\,(\frac{N}{T})^{N}} \qquad (7)$$

The Kupiec POF test has a chi-square distribution with one degree of freedom. If $\chi^2_{calculated} > \chi^2_{critical}$ the Null hypothesis is rejected and conclude that VaR model is not accurate and not consistent. If $\chi^2_{calculated} < \chi^2_{critical}$ the Null hypothesis is accepted thereby concluding that VaR model is accurate and consistent.

3.5 Testing for stationarity, normality, heteroscedasticity, and autocorrelation

Stationarity: The ADF test (also known as unit root or non-stationary test) is used to tests for stationarity of the return distribution

Normality: The Andersen Darling Test is used to test for normality of the monthly South African Industrial Index (J520) return distribution.

Heteroscedasticity: To test for the existence heteroscedasticity in residuals of monthly South Africa Industrial Index (J520) return distribution the Lagrange Multiplier (LM) test is used to test for the existence of Arch-effects.

Auto-correlation: Hakim, (2018), states that in order to apply the EVT method returns data has to be independent and identically distributed. To test for autocorrelation of the monthly South African Industrial Index (J520) data returns distribution the Ljung Test is used.

3.6 Data

This study employed the monthly South African Industrial Index (J520) return distribution accessed from iress expert: https://expert.inetbfa.com (with permission) for the years 1995-2018. These Indices are calculated from values of stocks based on industrial companies listed on the South African equity market respectively. These Indices estimate the overall performance of the equity market for a specific industry. There are currently a number of South African stock market Indices that are based on the South African Sector classification. Three broadly representative indices divide the South African equity market:

- SA Resources – Oil & Gas (J500) and Basic Materials (J510)
- SA Financials – Financials (J580) which include Banks, Insurance, Real Estate and Financial Services
- SA Industrials – Industrials (J520) which include Construction & Materials and Industrial Goods & Services)

Based on market capitalisation SA Resources currently account for around 12% of the South Africa's (SA) JSE All Share Index (ALSI), SA Industrials for 64% and SA Financials for 24%. In this study we modelled the monthly data of South African Industrial Index (J520) using the EVT from 1995 to 2018.

The data was transformed into monthly logarithmic returns as represented by the formula:

$$r_t = ln\ M_t/M_{t-1} \qquad (8)$$

where r_t is the monthly logarithmic returns at month t, M_t -the monthly returns at month t and ln - the natural logarithm.

To model the left tail we used the data as it is and for the right tail, the signs of the index returns data was changed so that r_t = - r_t.

4. Results and Discussion

The BM and the PoT methods, are used to predict the VaR in this study. The monthly returns of the South African Industrial Index (J520) are fitted to the BM method and PoT method over the period 1995–2018. The period chosen covers some major international financial crises. The researchers considered the right and the left tail of the Index returns. The data analysis uses R programming packages of fExtremes, extRemes, evir and ismev.

Table 1: Descriptive Statistics for monthly the South African Industrial returns Index (J520).

Description	Values
Number of Observations	271
Mean	0.009366
Median	0.010478
Minimum	-0.328471
Maximum	0.140273
Variance	0.003302
Standard Deviation	0.057467
Skewness	-1.016932
Kurtosis	4.420852

Source: Authors' own work.

The skewness and kurtosis are present in the return series, given in Table 1. The minima and maxima are far from the mean, which indicates the presence of extreme values. The skewness is negative which confirms the existence extreme values in the distribution. Kurtosis is greater than 3 which confirms that the return data is fat-tailed.

4.1 Testing for stationarity, normality, heteroscedasticity and autocorrelation results

Testing for Stationarity: The ADF Test is used to test for stationarity and the results show that a p-value < 0.05 was obtained:

Dickey-Fuller = -6.7391, Lag order = 6, p-value = 0.01

From the results it is concluded that the returns data is stationary

Test for Normality:

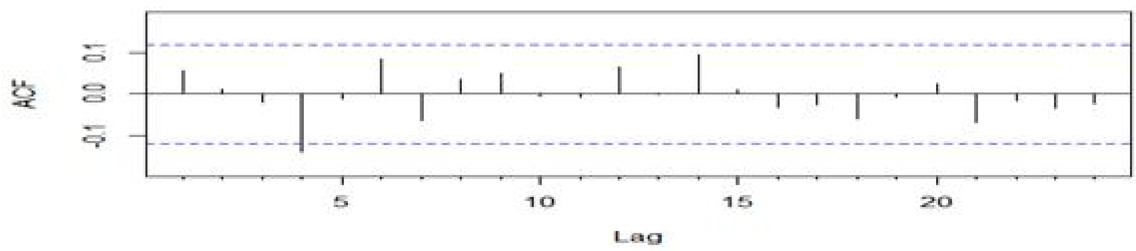
To test if the monthly South African Industrial Index (J520) data is normally distributed the Andersen Darlin g Normality Test is used. A p-value < 0.05 was obtained which implies that the monthly data series is not normally dis tributed. This suggests the returns data follow a fat-tailed distribution.

Test for Heteroscedasticity:

The Arch LM Test is used for the existence of ARCH effects in the monthly South African Industrial Index (J520) returns. The results indicate that there are no significant ARCH effects that exist in the returns data (χ^2 = 8.366974, df = 12, p-value = 0.7558355). Therefore, the heteroscedasticity of the monthly returns in this study are insignificant hence the adoption of the unconditional (static) models of the BM and PoT method.

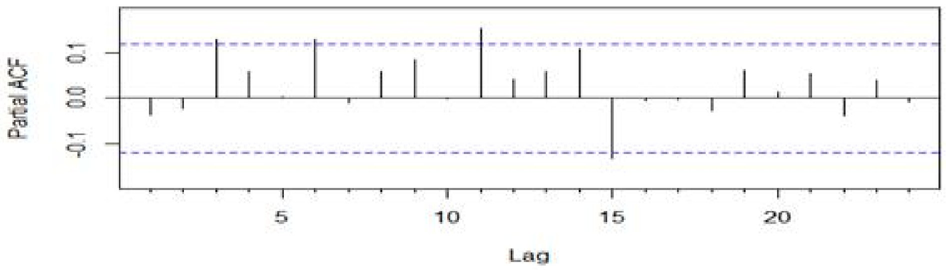
Test for Autocorrelation:

For the EVT methods to be applied, the returns data has to be independent and identically distributed. The ACF and PACF are used to check for autocorrelation.



Source: Authors' own work.

Figure 1: ACF diagram



Source: Authors' own work.

Figure 2: PACF Diagram

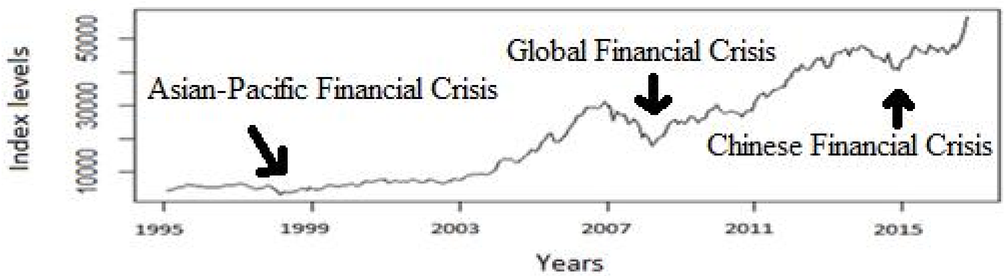
The ACF and PACF indicate that there are no significant auto correlations in the data

The Box-Ljung test for auto-correlation of the monthly South African Industrial Index (J520) return series was performed.

X-squared = 0.8806, df = 1, p-value = 0.348

The above test results show that there is no significant autocorrelation in the return distribution since the p-value > 0.05 was obtained. This implies that the return distribution is independently distributed.

4.2 Graphical plots of the South African Industrial Index (J520).

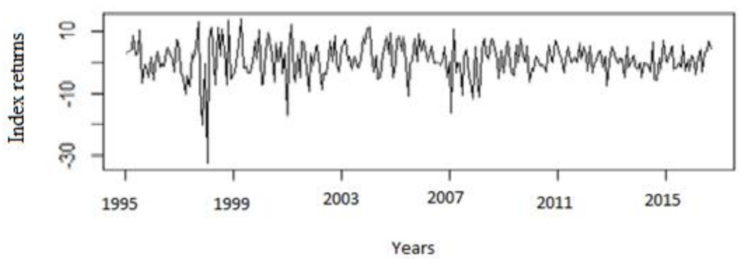


***Source*: Authors' own work.**

Figure 3: The monthly Index levels plot for the South African Industrial Index (J520).

The monthly Index levels plot shows a clear upward trend (Figure 3). The log return data is in Figure 4. The Asian-Pacific Financial Crisis (1997 to 1998), the Global Financial Crisis (2007 to 2008) and the Chinese Stock Market Crash. (2015 to 2016) had a negative impact on the SA equity market which are represented on the time series plot by sharp down turns of the SA Industrial index levels (Figure 3).

4.3 Time series plot of the log returns of the South African Industrial Index (J520)



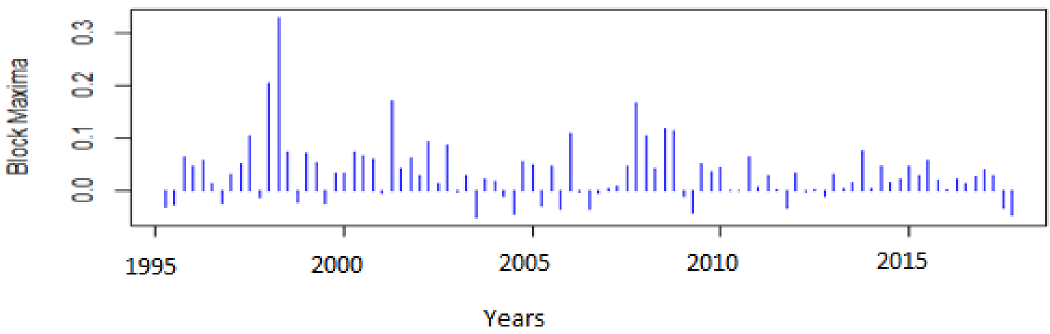
Source: Authors' own work.

Figure 4: The monthly log returns series plot of the South African Industrial Index (J520).

The log returns series in Figure 4 that show there is no presence of a unit root in the series as confirmed by the Augmented Dickey-Fuller (ADF) Test implying that the data is stationary.

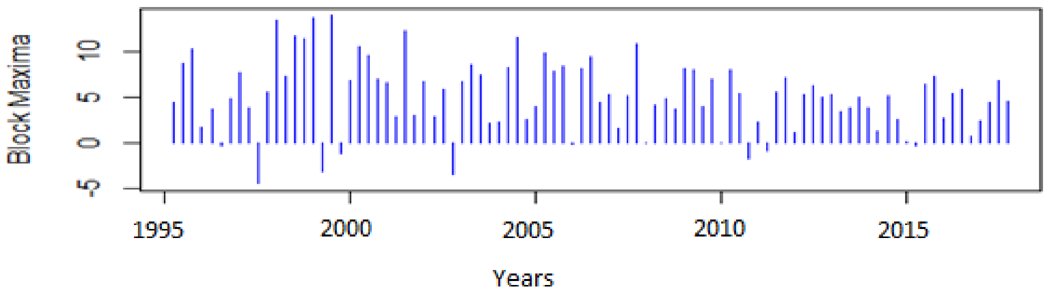
4.4 Generalised Extreme Value Distribution (GEVD)

The quarterly period minima/maxima was extracted from the South African Industrial Index (520) returns using the BM method, and then fitted to the GEVD. The negative and positive log-return maxima/minima were separated and fitted them separately to the GEVD.



***Source*: Authors' own work.**

Figure 5: The block minima (quarterly) for the negative return/ losses (right tail).

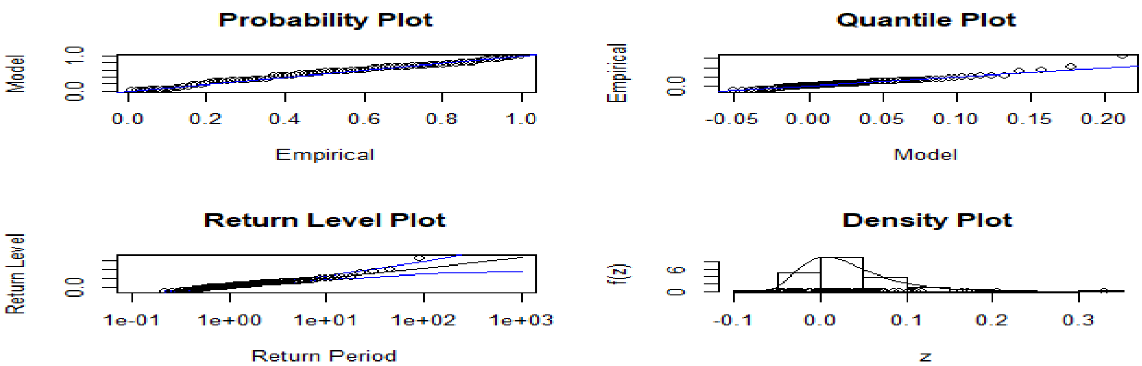


***Source*: Authors' own work.**

Figure 6: The block maxima (quarterly) for the positive returns/ gains (right tail).

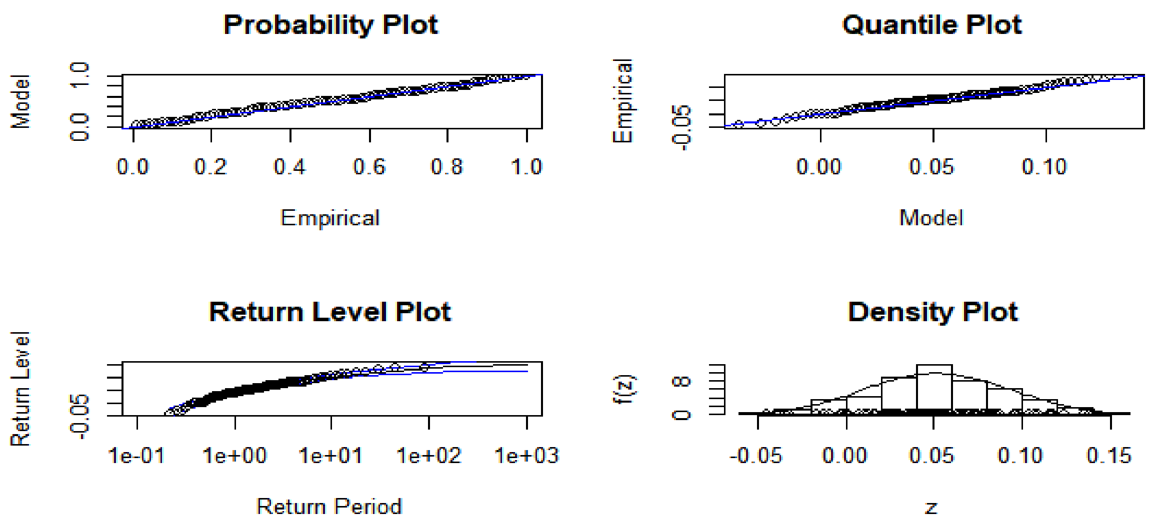
The maxima values were extracted using the BM method and fitted to the GEVD and their diagnostic plots are in Figure 7 and Figure 8.

4.5.0 Model diagnostics for the Block Maxima (BM) method



***Source*: Authors' own work.**

Figure 7: Model diagnostics for the right tail (negative returns/losses).



***Source*: Authors' own work.**

Figure 8: Model diagnostics for the left tail (positive returns/gains).

Figure 7 and Figure 8 show the diagnostic plots which indicate a goodness of fit for the right tail and left tail. The fitted models are used to calculate the parameters and VaR

4.5.1 Estimation of the tail distribution parameters using the BM method

Fitting the GEVD to the quarterly maxima/minima data of the BM method leads to the estimation of parameters and their related interval estimates.

Table 2: BM method parameters estimated using maximum likelihood estimate (MLE).

	Lower Bound	Point Estimate	Upper Bound
Right Tail (Losses, Negative returns or Minima)			
55% level of confidence			
Shape Parameter ξ	0.001604275	0.053584100	0.10556393
Scale Parameter σ	0.037431775	0.040007809	0.04287781
Location Parameter, u	0.004168446	0.007696946	0.01122545
Left Tail (Gains, Positive Returns or Maxima)			
95% level of confidence			
Shape parameter, ξ	-0.41848533	-0.29161190	-0.16473846
Scale Parameter, σ	0.03305153	0.03921980	0.04538806
Location Parameter, u	0.02950757	0.03836012	0.04721266

***Source*: Authors' own work.**

The extreme value index parameter for the right tail is not significant at very high confidence levels.

Table 2, shows the shape (ξ), scale ($\hat{\sigma}$), location (u) parameters: $\hat{\xi}$ = 0.053584100, $\hat{\sigma}$ = 0.040007809 and u = 0.007696946 respectively. Since $\xi > 0$ the quarterly interval model is a fat-tailed Fretchet class of distribution. Thi s implies the losses are unbounded and can be very big. The loss prospects are insignificant at higher levels of significa nce. However, the shape parameter is significant at 55 % level of confidence and the subsequent lower levels of signifi cance since their intervals do not include a zero as shown in Table 2, which implies that the loss prospects become sign ificant. This level is too uncomfortable to ignore.

The parameters and confidence intervals for the maxima (left tail) are in Table 2. The shape (ξ), scale ($\hat{\sigma}$) an d location (u) parameters estimated are $\hat{\xi}$ = -0.2916119, $\hat{\sigma}$ = 0.03921980 and u=0.03836012 respectively. This impli es that the gains (in the left tail) follow the short tailed negative Weibull class distribution since $\xi < 0$. This implies th e gains are upper bounded, meaning that they are limited. The shape parameter is negative at 95 % confidence level. T he negative shape parameter is significant because its interval does not include zero (see Table 2), hence the prospects of potential gains are significant.

4.5.2 Value at Risk (VaR) forecasts for the Block Maxima (BM) method

Table 3: BM method VaR forecast for the South African Industrial Index (J520).

Right tail (negative returns, losses)	
	Value at Risk (VaR)
Maximum potential loss	0.0867 (8.67%)
Left tail (positive returns, gains)	
Maximum potential gain	0.1156 (11.56%)

***Source*: Authors' own work.**

The results indicate that the possibility of potential losses (8.67%) is less than the possibility of potential gains (11.56%). This means that VaR forecast for the right tail is less than VaR forecast for the left tail. This implies that for one invested on the South African Industrial Index (J520), the possibility of potential losses is less than the possibility of potential gains.

4.6.0 Generalised Pareto Distribution (GPD)

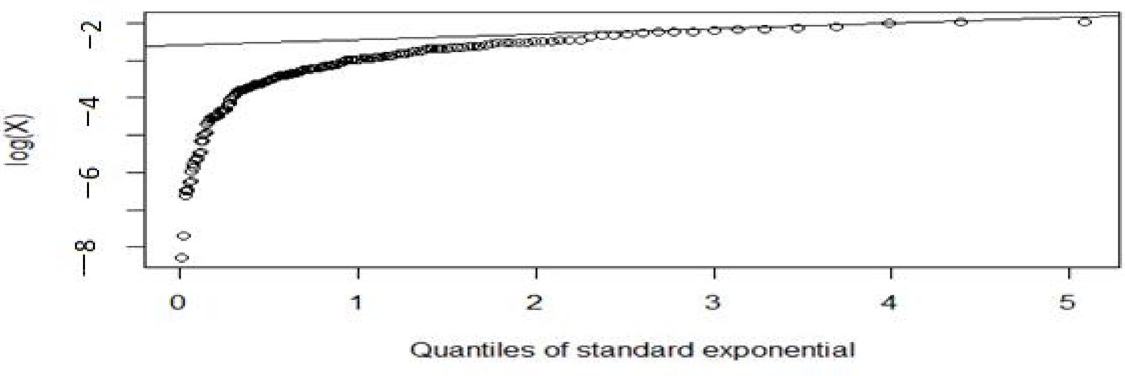
In this section we fit the South African Industrial Index (J520) returns to the GPD using the PoT method. The threshold was determined by Pareto QQ plot using the regression equation.

4.6.1 Threshold determination results

In this study the researchers considered both left and right tail. The data was therefore separated into two sets, right tail (negative returns/losses) and left tail (returns (positive returns/gains). The negative and positive returns were modelled separately using PoT method. The threshold was determined using the PoT method and the financial risk estimated in the form of VaR. The threshold for both the negative and positive returns were determined using the Pareto QQ plot.

4.6.2 The Pareto QQ Plot

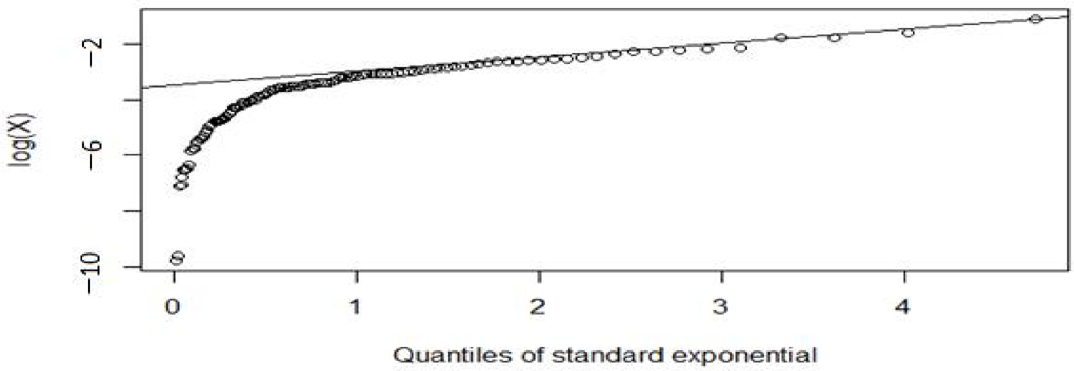
In this study, we used the Pareto QQ plot to estimate the thresholds.



***Source*: Authors' own work.**

Figure 9: Pareto QQ diagram for the right tail.

Using the regression line to the data points (Figure 9) as in (Sigauke, Makhwiting and Lesaoana (2014), we determined that it crosses the y-axis at -2.6 and therefore its exponent is 0.07 meaning that is the threshold estimate for the right tail (negative returns/losses).

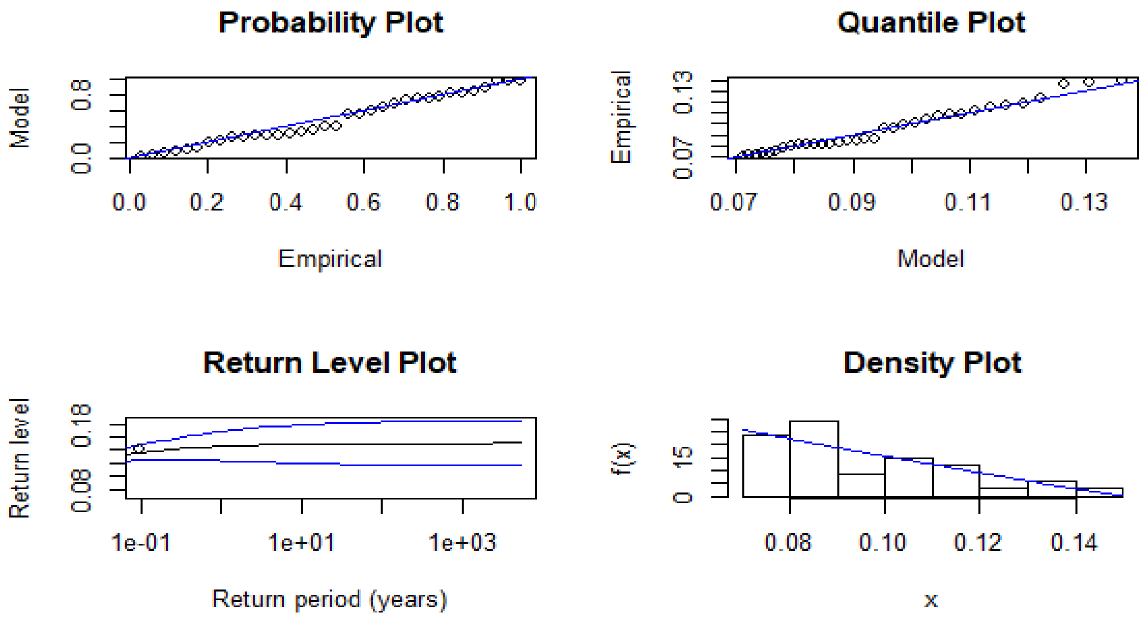


***Source*: Authors' own work.**

Figure 10: Pareto QQ diagram for the left tail.

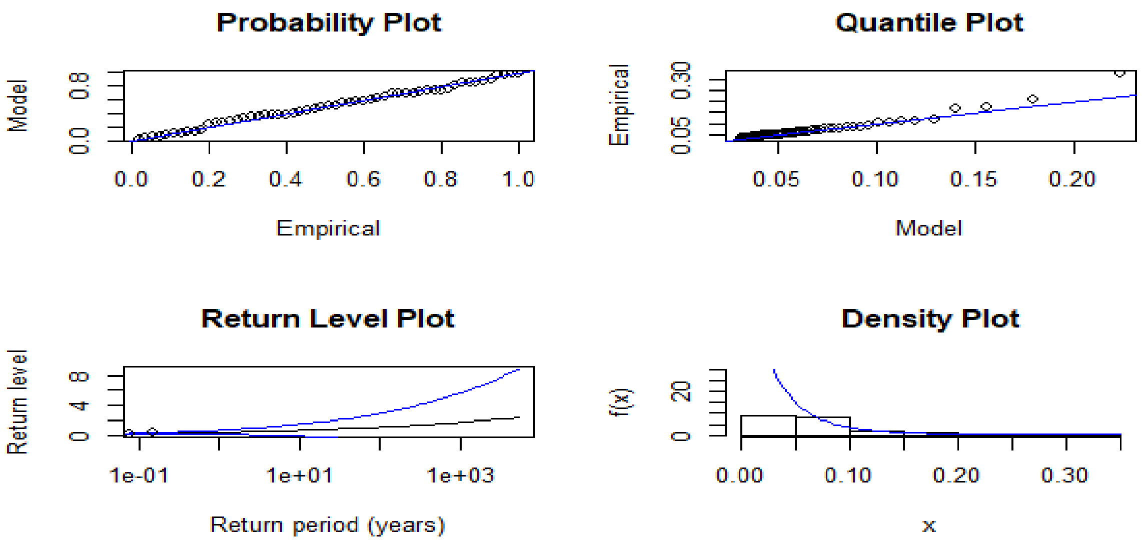
Using the regression line to the data points (Figure 10) as in (Sigauke, Makhwiting and Lesaoana (2014), we determined that it crosses the y-axis at -3.5 and therefore its exponent is 0.03. This is the threshold for the left tail (positive returns /gains).

4.6.3 Model Diagnostics for the Peak over Threshold (PoT) method



***Source*: Authors' own work.**

Figure 11: Diagnostic plots for the right tail (negative returns/losses).



***Source*: Authors' own work.**

Figure 12: Diagnostic plots for the left tail (positive returns/ gains).

The diagnostic plots in Figure 11 and Figure 12 indicate a goodness of fit performance for both the right and the left tail. The fitted models are used to estimate the parameters and VaR.

4.6.4 Estimation of the tail distribution parameters using the PoT method

Table 4: PoT method parameters estimated using maximum likelihood estimate (MLE).

	Right tail (Negative returns/losses)	Left tail (Positive returns/gains)
Threshold	u = 0.07	u = 0.03
Shape Parameter	-0.48207096	0.182889166
Standard error	0.22388147	0.149031473
Scale parameter	0.03894339	0.03262922
Standard error	0.01044674	0.006492347

***Source*: Authors' own work.**

The shape parameter for the left tail (gains), $\xi = 0.1829 > 0$, showing the distribution is fat-tailed. The shape parameter for the right tail (losses), $\xi = -0.4821 < 0$ showing the distribution belongs to the Weibull class. This means its upper bounded. The estimated tail parameters for the South African Industrial Index (J520) indicate the prospects of significant potential losses and significant potential gains which implies the Index returns are volatile. These findings have significant implications on how investors, researchers and practitioners should approach portfolio risk management. (for example, hedging, diversification and insurance)

4.6.5 Value at Risk (VaR) forecasts for the Peak over Threshold (PoT) method

Table 4: PoT method VaR forecast for the South African Industrial Index (J520).

Right tail (negative returns, losses)	
Value at Risk (VaR)	
Maximum potential loss	0.1317134 (13.17%)
Left tail (positive returns, gains)	
Value at Risk (VaR)	
Maximum potential gain	0.1602247 (16.02%)

***Source*: Authors' own work.**

The results indicate that the possibility of potential losses (13.17%) is less than the possibility of potential gains (16.02%). This means that the VaR forecast for the right tail is less than VaR forecast for the left tail. This implies that for one invested on the South African Industrial Index (J520), the possibility of potential losses is less than the possibility of potential gains.

4.7 Comparative analysis of BM method and PoT method VaR forecasts.

The BM method and PoT method VaR forecasts from Table 3 and Table 4 are tabulated in Table 5 for comparative purposes

Table 5: Comparative analysis of BM method and PoT method VaR results

Model	BM method	PoT method
	Negative Returns/Right tail return level (losses)	
Value-at-Risk	0.0867 (8.67%)	0.1317 (13.17%)
	Positive Returns/Left Tail Return Level (gains)	
Value-at-Risk	0.1156 (11.56%)	0.1602 (16.02%)

***Source*: Authors' own work.**

In Table 5 the estimated values of VaR for the BM method are less than the VaR for the PoT method. This is consistent with Sigauke, Makhwiting and Lesaoana (2014) who stated that estimates of extreme events provided by GEVD may underestimate the extreme events in some cases. The results in table 5 also indicate that for both the BM and PoT methods, the possibility of potential losses is less than the possibility of potential gains which is consistent with Nortey, Asare and Mettle (2016). Gilli and Këllezi (2006), found that possibility of potential losses is greater than the possibility of potential gains which is inconsistent with this study. The results from the estimated parameters show that the PoT method is the preferable model as it has a better goodness of fit performance (see Figure 7, Figure 8, Figure 11 and Figure 12). There are fewer deviations in the PoT method compared with the BM method, especially in the return level plots.

4.8 The performance of the BM and PoT methods evaluated using VaR forecasts.

In this test when the number of exceedances is known, the models are backtested using the Kupiec POF test to evaluate reliability and accuracy. The data used for backtesting is presented in Table 6:

Table 6: The evaluation of BM method and PoT method using backtesting.

Model	BM method VaR Exceedances	PoT method VaR Exceedances
	Negative Returns/Right tail return level (losses)	
Actual VaR exceedances	11	3
Expected VaR Exceedances	5	2
Total observations of extremes	91	34
Proportion of Failure	12.09%	8.82%
	Positive Returns/Left Tail Return Level (gains)	
Actual VaR exceedances	4	4
Expected VaR Exceedances	5	3
Total observations of extremes	91	55
Proportion of Failure	4.40%	7.27%

***Source*: Authors' own work.**

Table 7: Kupiec Proportion of failure (POF) test results

Kupiec POF Test			
	Test Statistic	Critical Value	Test Outcome
BM Right Tail	46.13	3.84	Reject
BM Left Tail	0.055	3.84	Accept
PoT Right Tail	0.861	3.84	Accept
PoT Left Tail	0.528	3.84	Accept

***Source*: Authors' own work.**

Backtesting evaluates the reliability and consistency of the VaR forecasts of BM and PoT methods. The number of exceedances and the empirical failure rate are presented in Table 6. The BM method right tail failed the test but the left tail passed the test. The PoT method passed the test for the two tails (see Table 7).

The findings reveal that the VaR forecasts based on the PoT method are more reliable and satisfactory than the BM method. PoT is more suitable when estimating VaR, and BM is more suitable when estimating return levels (Bucher and Zhou, 2018). The PoT method produces more efficient estimators than the BM method. The reason is that all large observations are used for the estimation of PoT estimators, while BM estimators may not utilize all the maxima/minima values in the same block therefore underestimation of VaR. The study concludes that the PoT method produces more reliable and satisfactory VaR forecasts than the BM method which is consistent with studies by Gilli and Kellezi (2006), Cerovic and Karadzic (2015), Bucher and Zhou (2018) and Szubzda and Chlebus (2019).

5. Conclusion and areas of further studies

This study investigated the performance of EVT models: the BM and the PoT methods are used to model the tail distribution of the South African Industrial Index (J520) over the period1995–2018. The return distribution is fitted to the BM and PoT methods. The BM and PoT methods provide a good fit for the right and left tails as confirmed by the diagnostic plots. Parameters were estimated for both methods using the MLE. The parameters are then used to estimate VaR for both methods.

The main findings of this study:

- the parameter estimations show that the PoT method is the preferable method as it has a better goodness of fit performance.
- the VaR forecasts for BM method are less than the VaR forecasts for the PoT method.
- the PoT method produces more reliable and satisfactory VaR forecasts than the BM method as validated by the Kupiec Test

The contribution of this study is that it supports the evidence that the PoT method produces more reliable and satisfactory VaR forecasts than the BM method.

The comparative analysis of the BM method and PoT methods with traditional methods which assume that financial returns are normally distributed would be an area interest of further study.

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